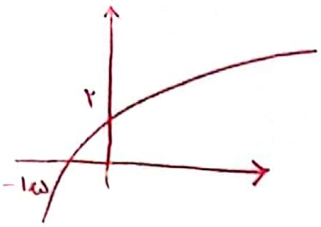
سؤال ۱)



$y = 1 - \log_c^{(ax-b)}$

b از I: $C = \frac{-1}{2}$

$\begin{pmatrix} -1 \\ 0 \end{pmatrix} \rightsquigarrow y = 1 - \log_c^{-1a-b} = 0 \rightarrow -1a - b = 1$

$\begin{pmatrix} 0 \\ 2 \end{pmatrix} \rightsquigarrow 1 - \log_c^{-b} = 2 \rightarrow c^{-1} = -b \Rightarrow \frac{1}{c} = -b$ II

I, II $\longrightarrow -C + \frac{1}{C} + \frac{3}{2} = 0 \quad -C^2 + \frac{3}{2}C + 1 = 0 \quad -2C^2 + 3C + 2 = 0 \quad C^2 + 3C - 4 = 0$

$(C+4)(C-1) = 0$

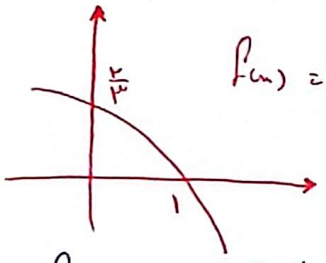
$\begin{cases} C = \frac{-4}{-2} = 2 \rightarrow -b = -\frac{1}{2} & \times \\ C = \frac{1}{-2} = \frac{-1}{2} \rightarrow -b = 2 & \checkmark \end{cases}$ $\boxed{b = -2}$ $\quad -b > 0$

$-2 + C = \frac{3}{2} \rightarrow \boxed{C = \frac{1}{2}}$

$1 - \log_{\frac{1}{2}}^{-1a+2} = 0 \quad \log_{\frac{1}{2}}^{-1a+2} = 1 \quad -1a + 2 = 0a$

$1a = 1a \rightarrow \boxed{a = 1} \quad -2(1a) = \boxed{-3}$

سؤال ۲)



$f(x) = 1 + C \times 3^{ax+b}$

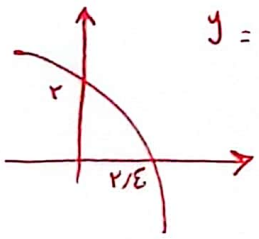
$\begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix} \rightsquigarrow 1 + C \times 3^{a} = \frac{2}{3} \Rightarrow C \times 3^{a} = -1 \times 3^{-1}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightsquigarrow 1 + 3^{b-1} = 0 \Rightarrow 3^{b-1} = 3^{0} \rightarrow b = 1$

$\xrightarrow{\text{جدا تساوی جمع}} \begin{cases} C = -1 \\ a = -1 \end{cases}$

$f(x) = 1 - 3^{x-1} \longrightarrow f(-1) = 1 - 3^{-2} = \boxed{\frac{8}{9}}$

سؤال ۳)



$y = c + \log_{\omega}^{(ax+b)} \qquad \frac{a}{b} = ?$

$c + \log_{\omega}^{b} = 2 \qquad \omega^{2-c} = b$

$c + \log_{\omega}^{2a+b} = 0 \qquad \omega^{-c} = 2a + b$

$\Big\} \rightarrow 2 \times \omega^{-c} = -2a$

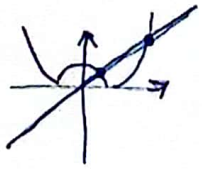
$2 \times \omega^{-c} = 2(\frac{-a}{10})$

$\xrightarrow{\text{جدا}} \begin{cases} C = 0 \\ a = -10 \end{cases}$

$\log_{\omega}^{-2+b} = 0 \quad -2 + b = 1 \quad \boxed{b = 2\omega}$

$\frac{a}{b} = \frac{-10}{2\omega} = \boxed{\frac{-5}{\omega}}$

سؤال ۴)



$f(x) = \log_{3}^{(|x^2-2|-x)}$

$|x^2 - 2| - x > 0$

$|x^2 - 2| > x$

$x^2 - 2 - x > 0 \rightarrow (x-2)(x+1) \quad x = 2 \checkmark \longrightarrow x > 2$ I صحیح نمودار

$-x^2 + 2 > x \quad x^2 + x - 2 < 0 \rightarrow (x+2)(x-1) \quad x = -2, 1$ II صحیح نمودار $\quad x < 1$

I, II $\longrightarrow x > 2 \ , \ x < 1 \quad (-\infty, 1) \cup (2, +\infty)$

سوال ۵)

$f(x) = 2 + 2^{b-ax}$

$g(x) = x^2 - 2x + 1$ ← طول نقطه تقاطع با محور yها

$f^{-1}(10) = -1 \rightarrow f(-1) = 10$

$2b - a = ?$

$g(1) = -(1)^2 - 2(1) + 1 = 4 \quad 2 + 2^{b-a} = 4 \quad 2^{b-a} = 2^1 \quad \begin{cases} b = a+1 \\ b+a = 2 \end{cases}$

$2 + 2^{b+a} = 10 \rightarrow 2^{b+a} = 2^3 \longrightarrow$

$2a = 2 \quad a = 1 \quad b = 2$

$2b - a = 2(2) - 1 = \boxed{3}$

سوال ۶)

$f(x) = -2 + \left(\frac{1}{2}\right)^{Ax+B}$ طول نقاط تقاطع $= 1, 2$ $\quad f(3) = ?$

$y = x^2 - x$

$y = (1)^2 - 1 = 0 \rightarrow -2 + 2^{-A-B} = 0 \quad 2^{-A-B} = 2^1 \longrightarrow -A - B = 1$

$y = (2)^2 - 2 = 2 \rightarrow -2 + 2^{-2A-B} = 2 \quad 2^{-2A-B} = 2^2 \longrightarrow -2A - B = 2$

$\Big\} \rightarrow -A = 1 \rightarrow A = -1 \quad B = 0$

سوال ۷)

$f(x) = -2 + 2^x = -2 + 2^3 = 6$ ← t ساعت

$\log_2^3 \simeq 1.6 \Rightarrow \frac{\log 3}{\log 2} = 1.6 \quad \log 2 \simeq \frac{\log 3}{1.6}$

$\log_3^4 \simeq 1.2 \Rightarrow \frac{\log 4}{\log 3} = 1.2 \quad \log 3 \simeq \frac{\log 4}{1.2}$

$M = M_0 \left(\frac{1}{8}\right)^t$ ← جرم اولیه، جرم باقی‌مانده

$\frac{1}{8} = \left(\frac{5}{8}\right)^t \quad \log_{10}^{8^{-1}} = t \log_{10}^{\frac{5}{8}}$

$-(\log 2 + \log 4) = t(3\log 2 - 2\log 4) \Rightarrow -\left(\frac{\log 3}{1.2} + \frac{\log 4}{1.2}\right) = t\left(\frac{3\log 3}{1.2} - \frac{2\log 4}{1.2}\right)$

$-\frac{3.8}{2.4 \times 1.2} = t\left(\frac{3.8 - 2.4}{2.4 \times 1.2}\right) \Leftarrow t = \frac{3.8}{1.5} \times 60 = 38$ دقیقه

سوال ۸)

$M = M_0 \left(\frac{7}{8}\right)^t$ ← t هفته

جرم باقی‌مانده ← جرم اولیه

$\left(\frac{1}{7}\right) = \left(\frac{7}{8}\right)^t \quad \log^{7^{-1}} = t \log^{\frac{7}{8}}$

$-\log^7 = t(\log^7 - 3\log^2)$

$\log_2^3 = \frac{\log^3}{\log 2} = 1.6 \rightarrow \log 2 = \frac{\log 3}{1.6}$

$-\frac{\log 3}{0.6} = t\left(\frac{\log 3}{0.6} - \frac{3\log 3}{1.6}\right)$

$\log_7^3 = \frac{\log^3}{\log 7} = 0.6 \rightarrow \log 7 = \frac{\log 3}{0.6}$

$\frac{-1.6}{9.6} = t\left(\frac{1.8 - 1.6}{9.6}\right) \Rightarrow t = 8$

$8 \times 7 = 56$ روز

سوال ۹)

روز اول: ۱۰۰٪ — روز دوم: ۱۰۰ − ۴ = ۹۶٪ — روز سوم: $96 - \frac{96}{100} \times 4 = 96(1 - \frac{4}{100})$

$100, 100(1-\frac{4}{100}), 100(1-\frac{4}{100})^2, \dots \longrightarrow$ روز $n+1$ ام: $100(1-\frac{4}{100})^n$ → بیت از دست رفته در روز n

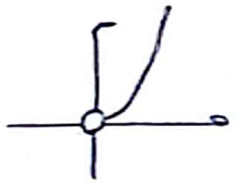
$100(1-\frac{4}{100})^n = \frac{1}{2} \times 100 \Rightarrow (\frac{24}{25})^n = \frac{1}{2}$

$$n = \log_{\frac{24}{25}} \frac{1}{2} = \frac{-\log 2}{\log 24 - \log 25} = \frac{-\log 2}{\log 3 + 3\log 2 - 2\log 5}$$

$$= \frac{-\log 2}{\log 3 + 3\log 2 - 2(1-\log 2)} = \frac{-\log 2}{\log 3 + 5\log 2 - 2} = \frac{-0.48}{0.48 + 0.7 - 2} = 4$$

سوال ۱۰)

الف) $y = 9^{\log_3 x} \longrightarrow x^{\log_3 9} = x^2$, $x > 0$



ب) $y = \log x^2 = \begin{cases} 2\log x & x > 0 \\ 2\log -x & x < 0 \end{cases}$