آزمون اینترنتی ۲۰ بنام خدا

سؤالهای تألیفی: ۲۶ گروه A

۱- (۲)

$S = \overbrace{3n \times 2n}^{\text{نسبت}} \times \sin 150 = 6n^2 \times \frac{1}{2} = 3n^2 = 63 \longrightarrow n^2 = 21 \rightarrow n = 3\sqrt{7}$

$P = 2(3n + 2n) = 10n = 10(3\sqrt{7}) = \boxed{30\sqrt{7}}$ ✓

۲- (۲)

$S_{\hat{ABC}} = \frac{1}{2} \times AB \times AC \times \sin A$ ← زاویه بین AC, AB

$\begin{cases} S_{\hat{ABC}} = \frac{1}{2} \times 8 \times V \times \sin A \\ S_{\hat{ADE}} = \frac{1}{2} \times 6 \times V \times \sin A \end{cases} \longrightarrow \Delta S = \frac{1}{2} \times V \times \sin A = 17V \rightarrow \sin A = \frac{1}{2}$

$\rightarrow \hat{A} = 30^\circ \rightarrow \tan A = \tan 30 = \boxed{\frac{\sqrt{3}}{3}}$ ✓

۳- (۲)

$\frac{1}{\sqrt{\cos^2 \alpha}} - \tan \alpha = \frac{1 + \sin \alpha}{|\cos \alpha|}$

$\frac{|\sin \alpha|}{\cos \alpha} = -\frac{1}{\cot \alpha}$

$\frac{|\sin \alpha|}{\cos \alpha} = -\frac{\sin \alpha}{\cos \alpha} \rightarrow |\sin \alpha| = -\sin \alpha$

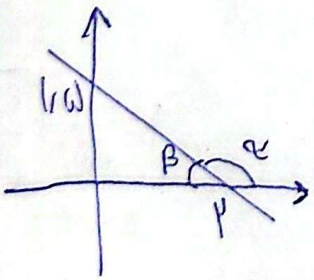
$\Rightarrow \sin \alpha < 0$

$\frac{1}{|\cos \alpha|} - \tan \alpha = \frac{1 + \sin \alpha}{|\cos \alpha|}$

$\rightarrow \tan \alpha = \frac{-\sin \alpha}{|\cos \alpha|} \rightarrow \frac{1}{\cos \alpha} = -\frac{1}{|\cos \alpha|} \rightarrow \cos \alpha < 0$

پس زاویه π ✓

۴- (۲)



$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha$

چون $\alpha + \beta = 180 \rightarrow \cot \alpha = -\cot \beta = -\frac{2}{10} = \boxed{-\frac{4}{3}}$ ✓

آرمین ایزدی

۵- (۲)

$$\frac{3\cos(2\cdot 18^\circ) - 2\sin(1\cdot 85^\circ)}{\sin(2\cdot 0^\circ) - \cos(2\cdot 9^\circ)} = \frac{-3\sin 22^\circ - 2\sin 22^\circ}{-\sin 22^\circ - \sin 22^\circ} = \frac{-5\sin 22^\circ}{-2\sin 22^\circ} = \frac{5}{2} = 2.5$$

($\cos(2\cdot 18^\circ) = -\sin 22^\circ$, $\sin(2\cdot 9^\circ) = \sin 22^\circ$)

۶- (۲)

$$\frac{\sin\left(\alpha + \frac{\pi}{2}\right) - \sin(\alpha - \pi)}{|\tan^2(\alpha) - 1|}$$

($\cos\alpha \leftarrow \sin\left(\alpha + \frac{\pi}{2}\right)$, $-\sin\alpha \leftarrow \sin(\alpha - \pi)$)

$\cos\alpha = \frac{2}{3}$

$$\sin^2\alpha = 1 - \cos^2\alpha = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\xrightarrow[\sin\alpha < 0]{\text{از}} -\frac{\sqrt{5}}{3} = \sin\alpha \Longrightarrow \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{\sqrt{5}}{3}}{\frac{2}{3}} = -\frac{\sqrt{5}}{2}$$

$$\rightarrow \frac{\cos\alpha + \sin\alpha}{|\tan^2\alpha - 1|} = \frac{\frac{2}{3} - \frac{\sqrt{5}}{3}}{\left|\frac{5}{4} - 1\right|} = \frac{\frac{2-\sqrt{5}}{3}}{\frac{1}{4}} = \frac{4(2-\sqrt{5})}{3} = \frac{8 - 4\sqrt{5}}{3}$$

۷- (۲)

$$\sin\alpha = 2\cos\alpha$$

$$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow 4\cos^2\alpha + \cos^2\alpha = 5\cos^2\alpha = 1 \rightarrow \cos\alpha = \frac{-\sqrt{5}}{5}$$

۸- (۲)

$$2mx + (m^2 - 1)y = 3 \longrightarrow a = -\frac{2m}{m^2 - 1} \quad (I)$$

$$\tan 60^\circ = \sqrt{3} \longrightarrow \left|\frac{2m}{m^2 - 1}\right| = \sqrt{3} \Rightarrow \sqrt{3}m^2 + 2m - \sqrt{3} = 0$$

$$\Delta = (2)^2 - 4(-\sqrt{3})(-\sqrt{3}) = 4 + 12 = 16$$

$$m_1 = \frac{-1+4}{2\sqrt{3}} = \frac{3}{2\sqrt{3}}$$

$$m_2 = \frac{-1-4}{2\sqrt{3}} = \frac{-3}{2\sqrt{3}}$$

$$\rightarrow |m_1 - m_2| = \left|\frac{5}{2\sqrt{3}}\right| = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

آمین ایزدی

۹- (۲)

$\tan(\frac{\pi}{4} - x) = \frac{1-m}{2+m}$ $\quad -\frac{\pi}{4} < x < \frac{\pi}{4} \rightarrow 0 < \frac{\pi}{4} - x < \frac{\pi}{2}$

↳ ربع اول $\rightarrow \tan > 0$

پس

$\Rightarrow \frac{1-m}{2+m} > 0 \rightarrow \boxed{-2 < m < 1}$ ✓

۱۰- (۲)

$\tan(300)\cos(210) + \tan(480)\sin(150) = -\tan 60 \times -\cos 30 + \overset{(-\cot 30)}{\tan 60} \times \cos 30$

$\rightarrow (-\sqrt{3})(-\frac{\sqrt{3}}{2}) + (-\sqrt{3})(\frac{\sqrt{3}}{2}) = \frac{3}{2} - \frac{3}{2} = 0$ ✓