آرمین ایزدی — بنام خدا — سؤالهای تألیفی: ۲۶ — گروه A

-۱

$S = \overbrace{۳n \times ۲n}^{\text{نسبت}} \times \sin ۱۵۰ = ۶n^۲ \times \frac{۱}{۲} = ۳n^۲ = ۵۴ \longrightarrow n^۲ = ۱۸ \rightarrow n = ۳\sqrt{۲}$

$P = ۲(۱۳n + ۲n) = ۱۵n = ۱۵(۳\sqrt{۲}) = \boxed{۴۵\sqrt{۲}}$

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$S_{\hat{ABC}} = \frac{۱}{۲} \times AB \times AC \times \sin A$ ← زاویه بین AC, AB

$\begin{cases} S_{\hat{ABC}} = \frac{۱}{۲} \times ۵ \times ۷ \times \sin A \\ S_{\hat{ADE}} = \frac{۱}{۲} \times ۲۰ \times ۷ \times \sin A \end{cases} \longrightarrow \Delta S = \frac{۱}{۲} \times ۷ \times \sin A = ۱۷۵ \rightarrow \sin A = \frac{۱}{۲}$

$\rightarrow \hat{A} = ۳۰^\circ \rightarrow \tan A = \tan ۳۰ = \boxed{\frac{\sqrt{۳}}{۳}}$

-۳

$\frac{۱}{\sqrt{\cos^۲ \alpha}} - \tan\alpha = \frac{۱ + \sin\alpha}{|\cos\alpha|}$ ⟵ $\frac{|\sin\alpha|}{\cos\alpha} = -\frac{۱}{\cot\alpha}$

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{\sin\alpha}{\cos\alpha} \rightarrow |\sin\alpha| = -\sin\alpha$

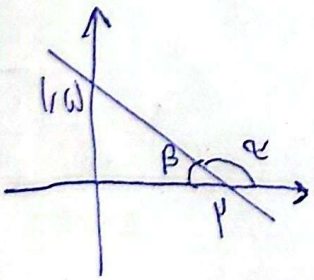
$\Rightarrow \sin\alpha < ۰$

$\frac{۱}{|\cos\alpha|} - \tan\alpha = \frac{۱ + \sin\alpha}{|\cos\alpha|}$

$\rightarrow \tan\alpha = \frac{-\sin\alpha}{|\cos\alpha|} \rightarrow \frac{۱}{\cos\alpha} = -\frac{۱}{|\cos\alpha|} \rightarrow \cos\alpha < ۰$

پس زاویه ۳

-۴



$\tan(\frac{\pi}{۲} - \alpha) = \cot\alpha$

چون $\alpha + \beta = ۱۸۰ \rightarrow \cot\alpha = -\cot\beta = -\frac{۳}{۱۰} = \boxed{-\frac{۴}{۳}}$

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-۵

$$\frac{3\cos(218^\circ) - 2\sin(135^\circ)}{\sin(210^\circ) - \cos(192^\circ)} = \frac{-3\sin 22^\circ - 2\sin 22^\circ}{-\sin 22^\circ - \sin 22^\circ} = \frac{-5\sin 22^\circ}{-2\sin 22^\circ} = \frac{5}{2} = 2.5$$

$\sin 22^\circ$, $-\sin 22^\circ$

-۶

$$\frac{\sin(\alpha + \frac{\pi}{2}) - \sin(\alpha - \pi)}{|\tan^2(\alpha) - 1|}$$

$\cos\alpha$ ↔ $-\sin\alpha$

$\cos\alpha = \frac{2}{3}$

$$\sin^2\alpha = 1 - \cos^2\alpha = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\xrightarrow[\sin\alpha < 0]{\text{از}} -\frac{\sqrt{5}}{3} = \sin\alpha \Longrightarrow \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{\sqrt{5}}{3}}{\frac{2}{3}} = -\frac{\sqrt{5}}{2}$$

$$\rightarrow \frac{\cos\alpha + \sin\alpha}{|\tan^2\alpha - 1|} = \frac{\frac{2}{3} - \frac{\sqrt{5}}{3}}{|\frac{5}{4} - 1|} = \frac{\frac{2 - \sqrt{5}}{3}}{\frac{1}{4}} = \frac{4(2 - \sqrt{5})}{3} = \frac{8 - 4\sqrt{5}}{3}$$

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$$\sin\alpha = 2\cos\alpha$$

$$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow 4\cos^2\alpha + \cos^2\alpha = 1 \rightarrow \cos\alpha = \frac{-\sqrt{5}}{5}$$

-۸

$$2mx + (m^2 - 1)y = 3 \xrightarrow{\text{شیب}} a = -\frac{2m}{m^2 - 1} \quad (I)$$

$$\tan 60^\circ = \sqrt{3} \longrightarrow \left|\frac{2m}{m^2 - 1}\right| = \sqrt{3} \Rightarrow \sqrt{3}m^2 + 2m - \sqrt{3} = 0$$

$$\Delta = (2)^2 - 4(-\sqrt{3})(-\sqrt{3}) = 4 + 12 = 16$$

$$m_1 = \frac{-1 + 4}{2\sqrt{3}} = \frac{3}{2\sqrt{3}}$$

$$m_2 = \frac{-1 - 4}{2\sqrt{3}} = \frac{-5}{2\sqrt{3}}$$

$$\rightarrow |m_1 - m_2| = \left|\frac{8}{2\sqrt{3}}\right| = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

آمین ایزدی

۹- $\tan\left(\frac{\pi}{4}-x\right)=\frac{1-m}{2+m}$ $\quad -\frac{\pi}{4}<x<\frac{\pi}{4} \rightarrow 0<\frac{\pi}{4}-x<\frac{\pi}{2}$

↳ ربع اول $\rightarrow \tan>0$

پس $\Rightarrow \frac{1-m}{2+m}>0 \rightarrow \boxed{-2<m<1}$

۱۰-

$\tan(300)\cos(210)+\tan(480)\sin(150)=-\tan 60 \times -\cos 30 + (-\cot 30) \times \cos 30$

$\rightarrow \left(-\sqrt{3}\right)\left(-\frac{\sqrt{3}}{2}\right)+\left(-\sqrt{3}\right)\left(\frac{\sqrt{3}}{2}\right)=\frac{3}{2}-\frac{3}{2}=0$