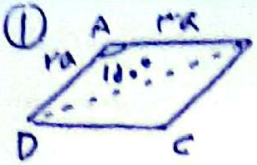
پاسخ تشریحی آزمون ۲۵ / پایه دهم ریاضی A

① 

$S_{ABCD} = 2S_{ABD} = 2 \times \frac{1}{2} \times AD \times AB \times \sin\hat{A}$ (I) ۲

$= 28 \times \frac{1}{2} \times 28 \times 28 \times \frac{1}{2} = 14a^2 = 196 \Rightarrow a^2 = 14 \Rightarrow a = 2\sqrt{7}$ (II) ۲

محیط متوازی الاضلاع $= 2(AD+AB) = 2(5a) = 10a \overset{(II)}{=} \boxed{20\sqrt{7}}$ ✓

$AD = BC$, $AB = DC$, $\hat{A} = \hat{C} = 150°$ $\Rightarrow \hat{ABD} \cong \hat{BCD}$

$\Rightarrow S_{\hat{ABD}} = S_{\hat{BCD}}$

$S_{ABCD} = S_{\hat{ABD}} + S_{\hat{BCD}}$ $\Rightarrow S_{ABCD} = 2S_{\hat{ABD}}$ (I)

② $S_{\hat{ABC}} = \frac{1}{2} AB \times AC \times \sin\hat{A} = \frac{1}{2} \times 5 \times 7 \times \sin\hat{A} = \frac{35}{4}\sin\hat{A}$

$S_{\hat{ADE}} = \frac{1}{2} AE \times AD \times \sin\hat{A} = \frac{1}{2} \times 2 \times 7 \times \sin\hat{A} = \frac{14}{4}\sin\hat{A}$

$\Rightarrow |S_{ABC} - S_{ADE}| = |\frac{21}{4}\sin\hat{A}| = 10.5$ ۲

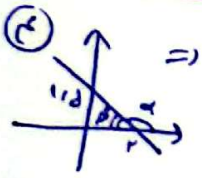
$\Rightarrow \begin{cases} \sin\hat{A} = \frac{1}{2} \Rightarrow \begin{cases} \hat{A} = 30° \checkmark \\ \hat{A} = 150° \Rightarrow \hat{A} < 90° \text{ غ ق ق} \end{cases} \\ \sin\hat{A} = -\frac{1}{2} \Rightarrow \text{غ ق ق} \end{cases}$ $\Rightarrow \boxed{\tan\hat{A} = \frac{\sqrt{3}}{3}}$ ✓

③ $\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1+\sin\alpha}{|\cos\alpha|} = \frac{1}{|\cos\alpha|} + \frac{\sin\alpha}{|\cos\alpha|}$ ۲

$\Rightarrow \frac{\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{-\cos\alpha} \Rightarrow |\cos\alpha| = -\cos\alpha \Rightarrow \boxed{\cos\alpha < 0}$ (I)

$\frac{|\sin\alpha|}{\cos\alpha} = -\tan\alpha = \frac{-\sin\alpha}{\cos\alpha} \Rightarrow |\sin\alpha| = -\sin\alpha \overset{(II)}{\Rightarrow} \boxed{\sin\alpha < 0}$

(I), (II) $\Rightarrow$ α در ناحیه سوم مثلثاتی قرار دارد ✓

④ 

$\tan\beta = \frac{11.5}{2} = \frac{23}{4}$ $\Rightarrow \tan(180° - \alpha) = \frac{23}{4} \Rightarrow \tan(\pi - \alpha) = \frac{23}{4} \Rightarrow \boxed{\tan\alpha = -\frac{23}{4}}$ ۲ (I)

$\beta = 180° - \alpha$

$\tan(\frac{\pi}{2} - \alpha) = \cot\alpha = \frac{1}{\tan\alpha} \overset{(I)}{=} \frac{1}{-\frac{23}{4}} = \boxed{-\frac{4}{23}}$ ✓

⑤ $\frac{2\cos(270° - 22°) - 2\sin(180° - 22°)}{\sin(180° + 22°) - \cos(270° + 22°)} = \frac{2\cos(\frac{3\pi}{2} - 22°) - 2\sin(\pi - 22°)}{\sin(\pi + 22°) - \cos(\frac{3\pi}{2} + 22°)} = \frac{-2\sin 22° - 2\sin 22°}{-\sin 22° - \sin 22°}$ ۲

$= \frac{-4\sin 22°}{-2\sin 22°} = \boxed{2}$ ✓

⑥ $\frac{\sin(\frac{\pi}{2} + \alpha) - \sin(\alpha - \pi)}{|\tan^2\alpha - 1|} = \frac{\cos\alpha + \sin\alpha}{|\tan^2\alpha - 1|} = \frac{\frac{2}{3} - \frac{\sqrt{5}}{3}}{|\frac{5}{4} - 1|} = \frac{\frac{2-\sqrt{5}}{3}}{\frac{1}{4}} = \boxed{\frac{8-4\sqrt{5}}{3}}$ ۲ ✓

$\cos\alpha = \frac{2}{3} \Rightarrow \cos^2\alpha = \frac{4}{9} \Rightarrow 1 - \cos^2\alpha = \sin^2\alpha = \frac{5}{9} \Rightarrow \sin\alpha = \pm\frac{\sqrt{5}}{3}$ (غ ق ق (α در ناحیه چهارم است)) $\Rightarrow \sin\alpha = -\frac{\sqrt{5}}{3}$

$\Rightarrow \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = -\frac{\sqrt{5}}{2}$

⑦ $\sin\alpha = 2\cos\alpha \Rightarrow \tan\alpha = 2 \Rightarrow 1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = 5 \Rightarrow \cos^2\alpha = \frac{1}{5} \Rightarrow \cos\alpha = \pm\frac{\sqrt{5}}{5}$ ۲

(غ ق ق (α در ناحیه سوم قرار دارد))

$\Rightarrow \boxed{\cos\alpha = -\frac{\sqrt{5}}{5}}$ ✓

⑧ $2mx + (m^2-1)y = 2 \Rightarrow y = \frac{-2m}{m^2-1}x + \frac{2}{m^2-1}$

شیب خط

شیب خط $= \tan 60° \Rightarrow \sqrt{3} = \frac{-2m}{m^2-1} \Rightarrow \sqrt{3}m^2 - \sqrt{3} = -2m \Rightarrow \sqrt{3}m^2 + 2m - \sqrt{3} = 0$ ۲

اختلاف مقادیر $m = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{4+12}}{\sqrt{3}} = \frac{4}{\sqrt{3}} = \boxed{\frac{4\sqrt{3}}{3}}$ ✓

⑨ $\tan\left(\frac{\pi}{4}-x\right)=\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\times\tan x}=\frac{1-\tan x}{1+\tan x}=\frac{1-m}{2+m}$

پاسخ تشریحی

$\Rightarrow \frac{1-\tan x+1+\tan x}{1+\tan x}=\frac{1-m+2+m}{2+m} \Rightarrow \frac{2}{1+\tan x}=\frac{3}{2+m} \Rightarrow \frac{1+\tan x}{2}=\frac{2+m}{3}$ (۲)

$\Rightarrow 1+\tan x=\frac{4+2m}{3} \Rightarrow \tan x=\frac{2m+1}{3}$

$-\frac{\pi}{4}<x<\frac{\pi}{4} \Rightarrow -1<\tan x<1$ $\Big\} \Rightarrow -1<\frac{2m+1}{3}<1 \Rightarrow -3<2m+1<3$

$\Rightarrow -4<2m<2 \Rightarrow \boxed{-2<m<1}$ ✓ (۲)

⑩ $\tan(200^\circ)\cos(110^\circ)+\tan(140^\circ)\sin(380^\circ)=\tan(200^\circ)\cos(110^\circ)+\tan(120^\circ)\sin(120^\circ)$

$=(-\sqrt{3})\left(-\frac{\sqrt{3}}{2}\right)+(-\sqrt{3})\left(\frac{\sqrt{3}}{2}\right)=\frac{3}{2}-\frac{3}{2}=\boxed{0}$ ✓