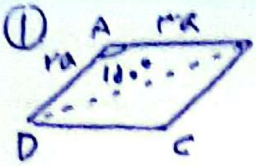
پارسا جهانگیری / یازدهم ریاضی A

①



$$\overset{(I)}{\Rightarrow} S_{ABCD} = 2S_{A\overset{\triangle}{B}D} = 2\times\frac{1}{2}\times AD\times AB\times \sin\hat{A}$$
$$= 2\times\frac{1}{2}\times 2a\times 3a\times\frac{1}{2} = 3a^2 = 54 \Rightarrow a^2 = 18 \Rightarrow a = 3\sqrt{2} \quad (II)$$

محیط متوازی الاضلاع $= 2(AD+AB) = 2(5a) = 10a \overset{(II)}{=} \boxed{30\sqrt{2}}$

$$\left.\begin{array}{l} AD = BC \\ AB = DC \\ \hat{A} = \hat{C} = 150^\circ \end{array}\right\} \Rightarrow A\overset{\triangle}{B}D \cong B\overset{\triangle}{C}D$$
$$\left.\begin{array}{l} \Rightarrow S_{A\overset{\triangle}{B}D} = S_{B\overset{\triangle}{C}D} \\ S_{ABCD} = S_{A\overset{\triangle}{B}D} + S_{B\overset{\triangle}{C}D} \end{array}\right\} \Rightarrow S_{ABCD} = 2S_{A\overset{\triangle}{B}D} \quad (I)$$

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②

$$\left.\begin{array}{l} S_{A\overset{\triangle}{B}C} = \frac{1}{2}AB\times AC\times\sin\hat{A} = \frac{1}{2}\times 5\times 7\times\sin\hat{A} = \frac{35}{2}\sin\hat{A} \\ S_{A\overset{\triangle}{D}E} = \frac{1}{2}AE\times AD\times\sin\hat{A} = \frac{1}{2}\times 4\times 7\times\sin\hat{A} = \frac{28}{2}\sin\hat{A} \end{array}\right\} \Rightarrow \left|S_{A\overset{\triangle}{B}C} - S_{A\overset{\triangle}{D}E}\right| = \left|\frac{7}{2}\sin\hat{A}\right| = 1.75$$

$$\Rightarrow \begin{cases} \sin\hat{A} = \frac{1}{2} \Rightarrow \begin{cases} \hat{A} = 30^\circ \checkmark \\ \hat{A} = 150^\circ \overset{\hat{A}<90^\circ}{\Longrightarrow} \text{غ ق ق} \end{cases} \\ \sin\hat{A} = -\frac{1}{2} \overset{\hat{A}<180^\circ}{\Longrightarrow} \text{غ ق ق} \end{cases} \overset{\hat{A}=30^\circ}{\Longrightarrow} \boxed{\tan\hat{A} = \frac{\sqrt{3}}{3}}$$

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③

$$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \underbrace{\cancel{\frac{1}{|\cos\alpha|}} - \frac{\sin\alpha}{\cos\alpha}} = \frac{1+\sin\alpha}{|\cos\alpha|} = \underbrace{\cancel{\frac{1}{|\cos\alpha|}} + \frac{\sin\alpha}{|\cos\alpha|}}$$

$$\Rightarrow \frac{\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{-\cos\alpha} \Rightarrow |\cos\alpha| = -\cos\alpha \Rightarrow \boxed{\cos\alpha < 0} \; (I)$$

$$\underbrace{\frac{|\sin\alpha|}{\cos\alpha}} = -\tan\alpha = \underbrace{\frac{-\sin\alpha}{\cos\alpha}} \Rightarrow |\sin\alpha| = -\sin\alpha \Rightarrow \boxed{\sin\alpha < 0} \; (II)$$

$\overset{(I),(II)}{\Longrightarrow}$ $\boxed{\text{α در ناحیه سوم مثلثاتی قرار دارد}}$

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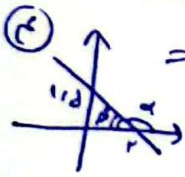
④



$$\left.\begin{array}{l} \Rightarrow \tan\beta = \frac{1.5}{2} = \frac{3}{4} \\ \beta = 180^\circ - \alpha \end{array}\right\} \Rightarrow \tan(180^\circ-\alpha) = \frac{3}{4} \Rightarrow \tan(\pi-\alpha) = \frac{3}{4} \Rightarrow \boxed{\tan\alpha = -\frac{3}{4}} \; (I)$$

$$\tan\left(\frac{\pi}{2}-\alpha\right) = \cot\alpha = \frac{1}{\tan\alpha} \overset{(I)}{=} \frac{1}{-\frac{3}{4}} = \boxed{-\frac{4}{3}}$$

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⑤

$$\frac{3\cos(270^\circ-22^\circ) - 2\sin(180^\circ-22^\circ)}{\sin(180^\circ+22^\circ) - \cos(270^\circ+22^\circ)} = \frac{3\cos(\frac{3\pi}{2}-22^\circ) - 2\sin(\pi-22^\circ)}{\sin(\pi+22^\circ) - \cos(\frac{3\pi}{2}+22^\circ)} = \frac{-3\sin22^\circ - 2\sin22^\circ}{-\sin22^\circ - \sin22^\circ}$$

$$= \frac{-5\sin22^\circ}{-2\sin22^\circ} = \boxed{\frac{5}{2}}$$

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⑥

$$\frac{\sin(\frac{\pi}{2}+\alpha) - \sin(\alpha-\pi)}{|\tan^2\alpha - 1|} = \frac{\cos\alpha + \sin\alpha}{|\tan^2\alpha - 1|} = \frac{\frac{2}{3} - \frac{\sqrt{5}}{3}}{|\frac{5}{4}-1|} = \frac{\frac{2-\sqrt{5}}{3}}{\frac{1}{4}} = \boxed{\frac{8-4\sqrt{5}}{3}}$$

غ ق ق (α در ناحیه چهارم قرار دارد)

$$\cos\alpha = \frac{2}{3} \Rightarrow \cos^2\alpha = \frac{4}{9} \Rightarrow 1-\cos^2\alpha = \sin^2\alpha = \frac{5}{9} \Rightarrow \sin\alpha = \pm\frac{\sqrt{5}}{3} \Rightarrow \sin\alpha = -\frac{\sqrt{5}}{3}$$

$$\Rightarrow \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = -\frac{\sqrt{5}}{2}$$

غ ق ق (α در ناحیه سوم قرار دارد)

⑦ $\sin\alpha = 2\cos\alpha \Rightarrow \tan\alpha = 2 \Rightarrow 1+\tan^2\alpha = \frac{1}{\cos^2\alpha} = 5 \Rightarrow \cos^2\alpha = \frac{1}{5} \Rightarrow \cos\alpha = \pm\frac{\sqrt{5}}{5}$

$\Rightarrow \boxed{\cos\alpha = -\frac{\sqrt{5}}{5}}$

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⑧ $2mx + (m^2-1)y = 3 \Rightarrow y = \overset{\text{شیب خط}}{\boxed{\frac{-2m}{m^2-1}}}\,x + \frac{3}{m^2-1}$

شیب خط $= \tan60^\circ \Rightarrow \sqrt{3} = \frac{-2m}{m^2-1} \Rightarrow \sqrt{3}m^2 - \sqrt{3} = -2m \Rightarrow \sqrt{3}m^2 + 2m - \sqrt{3} = 0$

اختلاف مقادیر m $= \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{4+12}}{\sqrt{3}} = \frac{4}{\sqrt{3}} = \boxed{\frac{4\sqrt{3}}{3}}$

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(9) $\tan(\frac{\pi}{4} - x) = \dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \times \tan x} = \dfrac{1 - \tan x}{1 + \tan x} = \dfrac{1-m}{2+m}$

ترکیب مخرج در صورت

$\Rightarrow \dfrac{1 - \tan x + 1 + \tan x}{1 + \tan x} = \dfrac{1 - m + 2 + m}{2+m} \Rightarrow \dfrac{2}{1+\tan x} = \dfrac{3}{2+m} \Rightarrow \dfrac{1+\tan x}{2} = \dfrac{2+m}{3}$

$\left.\begin{array}{l} \Rightarrow 1 + \tan x = \dfrac{4+2m}{3} \Rightarrow \tan x = \dfrac{2m+1}{3} \\ -\frac{\pi}{4} < x < \frac{\pi}{4} \Rightarrow -1 < \tan x < 1 \end{array}\right\} \Rightarrow -1 < \dfrac{2m+1}{3} < 1 \Rightarrow -3 < 2m+1 < 3$

$\Rightarrow -4 < 2m < 2 \Rightarrow \boxed{-2 < m < 1}$

(10) $\tan(300°)\cos(210°) + \tan(480°)\sin(840°) = \tan(300°)\cos(210°) + \tan(120°)\sin(120°)$

$= (-\sqrt{3})(-\frac{\sqrt{3}}{2}) + (-\sqrt{3})(\frac{\sqrt{3}}{2}) = \frac{3}{2} - \frac{3}{2} = \boxed{0}$