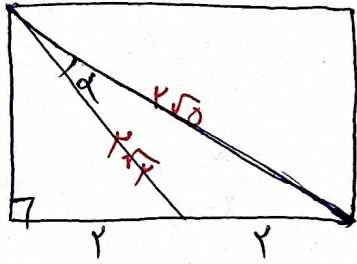


$$S = \frac{1}{2} (\sqrt{3}) (2) (\sin \alpha) = 1,8 \rightarrow \alpha = \frac{\sqrt{3}}{2} \Rightarrow \begin{cases} \alpha = 40^\circ \text{ در } \min \\ \alpha = 140^\circ \text{ در } \max \end{cases}$$

$$\Rightarrow \frac{1,8}{40} = \frac{2}{100}$$

min
max
1

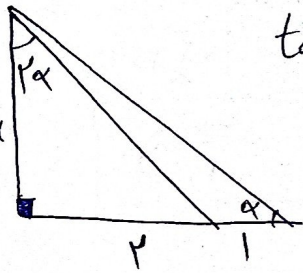


$$2 = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} - 2\sqrt{2} \cos \alpha \rightarrow 2\sqrt{2} - (2\sqrt{2}) \cos \alpha = 2$$

$$\rightarrow \cos \alpha = \frac{2\sqrt{2}}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2} \xrightarrow{\text{زاویه } \alpha} \sin \alpha = \sqrt{1 - \frac{2}{4}} = \frac{\sqrt{2}}{2}$$

$$\rightarrow \cot \alpha = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

2
3



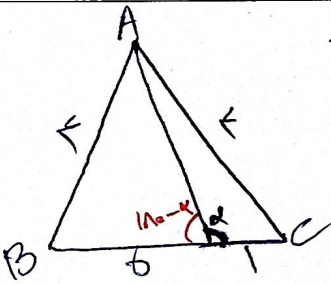
$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \rightarrow \cot 2\alpha = \frac{1 - \tan^2 \alpha}{2 \tan \alpha} \quad \tan \alpha = \frac{x}{1}$$

$$\cot 2\alpha = \frac{x}{1} = \frac{1 - \tan^2 \alpha}{2 \tan \alpha} \xrightarrow{\tan \alpha = \frac{x}{1}} \frac{x}{1} = \frac{1 - \frac{x^2}{1}}{\frac{2x}{1}} \Rightarrow$$

$$x^2 = 1 - \frac{x^2}{1} \rightarrow \frac{2}{1} x^2 = 1 \rightarrow x = \pm \frac{1}{\sqrt{2}} \rightarrow \boxed{x = \frac{1}{\sqrt{2}}}$$

$$\cot \alpha = \frac{1}{x} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

3
4



$$\tan \alpha = -\tan(180^\circ - \alpha)$$

$$|AH|^2 = 14 - 9 = 5 \rightarrow AH = \sqrt{5}$$

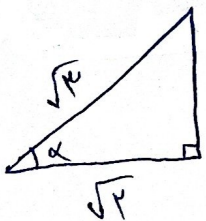
$$\tan \beta = \frac{\sqrt{5}}{1} \rightarrow \tan \alpha = \frac{\sqrt{5}}{1}$$



$$\beta = 180^\circ - \alpha$$

4
5

$$\sin^2 \alpha + \cos^2 \alpha + \sin^2 \alpha = \frac{5}{1} \Rightarrow \sin^2 \alpha = \frac{1}{1} \rightarrow \sin \alpha = \pm \frac{1}{\sqrt{2}}$$



$$\Rightarrow \tan \alpha = \pm \frac{1}{\sqrt{2}} \rightarrow \tan^2 \alpha = \frac{1}{2}$$

5

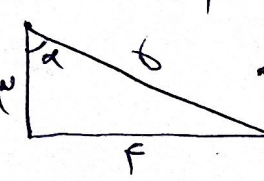
$$\frac{\sin^2 \alpha + \cos^2 \alpha}{1 + \cos^2 \alpha} - \frac{\cos^2 \alpha - \sin^2 \alpha}{1 + \sin^2 \alpha} \Rightarrow \begin{cases} \sin^2 \alpha = (\sin^2 \alpha)^2 = (1 - \cos^2 \alpha)^2 = \cos^4 \alpha - 2\cos^2 \alpha + 1 \\ \cos^2 \alpha = (\cos^2 \alpha)^2 = (1 - \sin^2 \alpha)^2 = \sin^4 \alpha - 2\sin^2 \alpha + 1 \end{cases}$$

$$\Rightarrow \frac{(\cos^4 \alpha - 2\cos^2 \alpha + 1) + \cos^2 \alpha}{1 + \cos^2 \alpha} - \frac{(\sin^4 \alpha - 2\sin^2 \alpha + 1) + \sin^2 \alpha}{1 + \sin^2 \alpha} = \frac{(\cos^2 \alpha + 1)^2}{\cos^2 \alpha + 1} \cdot \frac{(\sin^2 \alpha + 1)^2}{\sin^2 \alpha + 1}$$

$$\Rightarrow \cos^2 \alpha + 1 - \sin^2 \alpha - 1 = \cos^2 \alpha - \sin^2 \alpha \Rightarrow \cos 2\alpha$$

$$(\cos \alpha)(-\sin \alpha) - (-\cot \alpha) = -(\cos \alpha)(\sin \alpha) + \cot \alpha$$

$\tan \alpha = \frac{F}{P}$



$\sin \alpha = \frac{F}{\sqrt{F^2 + P^2}}$
 $\cos \alpha = \frac{P}{\sqrt{F^2 + P^2}}$
 $\cot \alpha = \frac{P}{F}$

$$\left(\frac{P}{\sqrt{F^2 + P^2}}\right)\left(\frac{F}{\sqrt{F^2 + P^2}}\right) + \frac{P}{F} = \frac{-FP}{F^2 + P^2} + \frac{P}{F} = \frac{-FP + P(F^2 + P^2)}{F(F^2 + P^2)} = \frac{PF^2 + P^3 - FP}{F(F^2 + P^2)} = \frac{PF^2 + P^3 - FP}{100} = \frac{PV}{100}$$

$$\sin \frac{\pi}{14} = \sin 18^\circ = \sqrt{\frac{1 - \cos 36^\circ}{2}} = \sqrt{\frac{1 - \frac{\sqrt{5}-1}{4}}{2}} = \frac{\sqrt{2 - \sqrt{5}}}{2}$$

$$\cos \frac{\pi}{14} = \cos 18^\circ = \sqrt{\frac{1 + \cos 36^\circ}{2}} = \sqrt{\frac{1 + \frac{\sqrt{5}-1}{4}}{2}} = \frac{\sqrt{2 + \sqrt{5}}}{2}$$

$$\frac{P}{F} \cos 36^\circ + \sqrt{2} \left(\frac{\sqrt{2 - \sqrt{5}}}{2} \right) - \sqrt{2} \left(\frac{\sqrt{2 + \sqrt{5}}}{2} \right) = \frac{P}{F} + \frac{\sqrt{2}(\sqrt{2 - \sqrt{5}} - \sqrt{2 + \sqrt{5}})}{2}$$

$$\Rightarrow \frac{P}{F} + \frac{(-2)}{2} = \frac{1}{2}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \Rightarrow \tan \alpha = \frac{2 \tan(\frac{\alpha}{2})}{1 - \tan^2(\frac{\alpha}{2})} = \frac{1}{\frac{1}{2}} = \frac{1}{\frac{1}{14}} = \frac{14}{1} = \frac{14}{10} = \frac{7}{5}$$

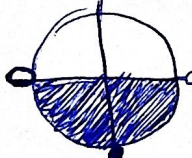
$\tan \frac{\alpha}{2} = \frac{1}{2} \rightarrow \tan \alpha > 0 \rightarrow \alpha = \text{دو ربعی}$

$$\sin 2\alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha} \Rightarrow \sin \alpha = \frac{2 \tan(\frac{\alpha}{2})}{1 + \tan^2(\frac{\alpha}{2})} = \frac{1}{1 + \frac{1}{4}} = \frac{4}{5}$$

$$\cos 2\alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha} \Rightarrow \cos \alpha = \frac{1 - \tan^2(\frac{\alpha}{2})}{1 + \tan^2(\frac{\alpha}{2})} = \frac{10}{14} \Rightarrow \frac{\tan \alpha - \sin \alpha}{\sin \alpha - \cos \alpha} = \frac{\frac{7}{5} - \frac{4}{5}}{\frac{4}{5} - \frac{10}{14}} = \frac{-1}{10}$$

$\sin^2 \alpha = 2 \sin \alpha \cos \alpha \rightarrow 2 \sin \alpha < \sin^2 \alpha \rightarrow \sin \alpha < \sin \alpha \cos \alpha$

این که $\sin \alpha$ و $\cos \alpha$ هر دو بین 0 و 1 هستند و ضرب آن دو بین 0 و 1 است. از آنجا که $\sin \alpha$ کوچکتر از $\sin \alpha \cos \alpha$ است، حاصل عبارت از حرکت از آن دو عددی کوچکتر است.



$$\frac{\cot \alpha}{\sin \alpha} = \frac{\cos \alpha}{\sin^2 \alpha} \rightarrow \cos > 0 \Rightarrow \alpha = \text{دو ربعی}$$