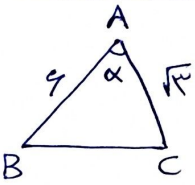
-۱

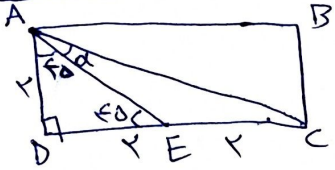


$$S=\frac{1}{2}\times 6\times\sqrt{43}\times\sin\alpha=3\sqrt{43}\sin\alpha=24,5$$

$$\Rightarrow \sin\alpha=\frac{24,5}{3\sqrt{43}}=\frac{3}{2\sqrt{3}}=\frac{\sqrt{3}}{2}$$

$$0<\alpha<180 \longrightarrow \begin{cases}\alpha_{max}=120^\circ\\ \alpha_{min}=60^\circ\end{cases}\Rightarrow \frac{Max}{Min}=\frac{120}{60}=2$$

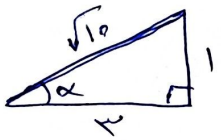
-۲



$$AE=\sqrt{2^2+2^2}=2\sqrt{2}\qquad AC=\sqrt{2^2+6^2}=2\sqrt{10}$$

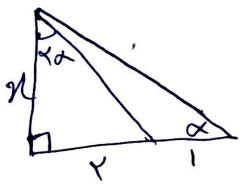
$$EC=\sqrt{a^2+b^2-2ab\cos\alpha}=\sqrt{8+40-8\sqrt{10}\cos\alpha}=2\sqrt{6}$$

$$\Rightarrow \sqrt{48-8\sqrt{10}\cos\alpha}=\sqrt{6}\Rightarrow 48-8\sqrt{10}\cos\alpha=6\Rightarrow\cos\alpha=\frac{3}{\sqrt{10}}$$



$$\cot\alpha=3$$

-۳

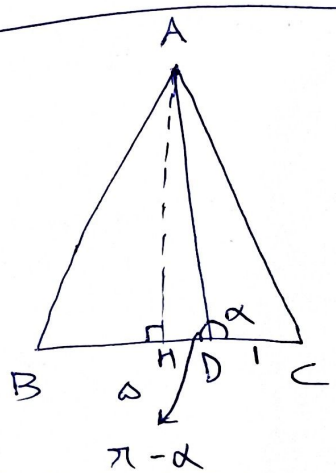


$$\tan\alpha=\frac{x}{2}\qquad \cot 2\alpha=\frac{x}{2}$$

$$\cot 2\alpha=\frac{1-\tan^2\alpha}{2\tan\alpha}=\frac{1-\frac{x^2}{9}}{\frac{2x}{3}}=\frac{x}{2}$$

$$1-\frac{x^2}{9}=\frac{x^2}{3}\Rightarrow\frac{4x^2}{9}=1\Rightarrow x=\frac{3}{2}\Rightarrow\cot\alpha=\frac{3}{x}=\frac{3}{\frac{3}{2}}=2$$

-۴



$$AH=\sqrt{AB^2-BH^2}=\sqrt{16-9}=\sqrt{7}$$

$$\tan(\pi-\alpha)=\frac{AH}{HD}=\frac{\sqrt{7}}{2}$$

$$\tan\alpha=-\tan(\pi-\alpha)=-\frac{\sqrt{7}}{2}$$

-۵

$$\sin^2 x+\underbrace{\sin^2 x+\cos^2 x}_{1}=\frac{4}{3}\Rightarrow\sin^2 x=\frac{1}{3}$$

$$\Rightarrow \frac{1}{3}+\cos^2 x=1\Rightarrow\cos^2 x=\frac{2}{3}$$

$$\tan^2 x=\frac{\sin^2 x}{\cos^2 x}=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{2}$$

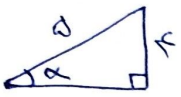
۶- $\sin^۴\alpha = (\sin^۲\alpha)^۲ = (۱-\cos^۲\alpha)^۲ = \cos^۴\alpha + ۱ - ۲\cos^۲\alpha$

$\cos^۴\alpha = (\cos^۲\alpha)^۲ = (۱-\sin^۲\alpha)^۲ = \sin^۴\alpha + ۱ - ۲\sin^۲\alpha$

$$\frac{\cos^۴\alpha + ۱ - ۲\cos^۲\alpha + ۴\cos^۲\alpha}{۱+\cos^۲\alpha} - \frac{\sin^۴\alpha + ۱ - ۲\sin^۲\alpha}{۱+\sin^۲\alpha}$$

$$= \frac{(\cos^۲\alpha+۱)^۲}{\cos^۲\alpha+۱} - \frac{(\sin^۲\alpha+۱)^۲}{\sin^۲\alpha+۱} = \cos^۲\alpha + ۱ - \sin^۲\alpha - ۱ = \cos^۲\alpha - \sin^۲\alpha = \boxed{\cos ۲\alpha}$$

۷-



$\sin\alpha = -\frac{۴}{۵}$ $\quad \cot\alpha = \frac{۳}{۴}$

$\cos\alpha = -\frac{۳}{۵}$

$\sin\left(\frac{۹\pi}{۲}+\alpha\right) = \cos\alpha = -\frac{۳}{۵}$ $\quad \cos\left(\frac{۷\pi}{۲}-\alpha\right) = -\sin\alpha = \frac{۴}{۵}$

$\tan\left(\alpha - \frac{۳\pi}{۲}\right) = -\cot\alpha = -\frac{۳}{۴}$

$\sin\left(\frac{۹\pi}{۲}+\alpha\right)\cos\left(\frac{۷\pi}{۲}-\alpha\right) - \tan\left(\alpha - \frac{۳\pi}{۲}\right) = \left(-\frac{۳}{۵}\right)\times\frac{۴}{۵} - \left(-\frac{۳}{۴}\right)$

$= \frac{-۴۸+۷۵}{۱۰۰} = \boxed{\frac{۲۷}{۱۰۰}}$

۸-

$۲\cos ۴\pi = ۲\cos\frac{\pi}{۳} = ۲\times\frac{۱}{۲} = \frac{۳}{۲}$

$\sqrt{۲}\sin\pi - \sqrt{۲}\cos\pi = \sqrt{۲}(\sin\pi - \cos\pi)$

$\xrightarrow{\text{به توان ۲}}$ $۲(\sin^۲\pi + \cos^۲\pi - ۲\sin\pi\cos\pi) = ۲(۱-\sin^۲\pi)$

$= ۲ - ۲\sin\frac{\pi}{۶} = ۲ - ۱ = ۱ \Longrightarrow \sqrt{۲}\sin\pi - \sqrt{۲}\cos\pi = \begin{cases} ۱ & \checkmark \\ -۱ & \times \end{cases}$

$\Longrightarrow \frac{۳}{۲} + (-۱) = \boxed{\frac{۱}{۲}}$

$\sin\pi - \cos\pi < ۰ \Leftarrow \pi = \frac{\pi}{۱۴}$

۹-

$\frac{\sin\frac{\alpha}{۲}}{\cos\frac{\alpha}{۲}} = \frac{۱}{۴} \Rightarrow \cos\frac{\alpha}{۲} = ۴\sin\frac{\alpha}{۲}$ $\quad \sin^۲\frac{\alpha}{۲} + \cos^۲\frac{\alpha}{۲} = ۱$

$\Rightarrow \sin^۲\frac{\alpha}{۲} + ۱۶\sin^۲\frac{\alpha}{۲} = ۱ \Rightarrow \sin\frac{\alpha}{۲} = \frac{۱}{\sqrt{۱۷}}$

$\cos\frac{\alpha}{۲} = \frac{۴}{\sqrt{۱۷}}$

$\sin\alpha = ۲\sin\frac{\alpha}{۲}\cos\frac{\alpha}{۲} = ۲\times\frac{۱}{\sqrt{۱۷}}\times\frac{۴}{\sqrt{۱۷}} = \frac{۸}{۱۷}$

$\cos\alpha = \cos^۲\frac{\alpha}{۲} - \sin^۲\frac{\alpha}{۲} = \frac{۱۶}{۱۷} - \frac{۱}{۱۷} = \frac{۱۵}{۱۷}$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{۸}{۱۵} - \frac{۸}{۱۷}}{-\frac{۷}{۱۷}} = \boxed{-\frac{۱۶}{۱۰۵}}$

۱۰-

$\sin ۲\alpha = ۲\sin\alpha\cos\alpha \Longrightarrow ۲\sin\alpha < ۲\sin\alpha\cos\alpha$

$\Rightarrow ۲\sin\alpha\cos\alpha - ۲\sin\alpha > ۰$ $\quad ۲\sin\alpha(\cos\alpha - ۱) > ۰ \Rightarrow \sin\alpha < ۰$

$\frac{\cot\alpha}{\sin\alpha} > ۰ \Longrightarrow \cot\alpha < ۰ \Longrightarrow$ (منفی) $\cos\alpha > ۰ \Rightarrow$ ناحیه چهارم