«باسمه تعالی»

نام و نام خانوادگی: علی ربانی ۲۰ پاسخ تکلیف شماره ۲۷ کلاس یازدهم ۸۰

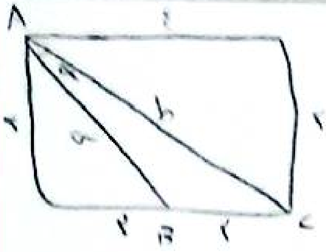
۱- (۲)

$S = \frac{1}{2} a b \sin\alpha = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin\alpha = 1{,}5$

$\sqrt{3} \times \sin\alpha = 1{,}5$

$\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \quad \alpha \begin{cases} 120^\circ \\ 60^\circ \end{cases} \Rightarrow \frac{120}{60} = \boxed{2}$ ✓

۲- (۲)



$S_{ABC} = \frac{1}{2} \times 2 \times 2 = 2 = \frac{1}{2} a b \sin\alpha$

$\frac{1}{2} \times \sqrt{20} \times \sqrt{8} \times \sin\alpha = 2$

$\sqrt{160} \times \sin\alpha = 4$

$\cot\alpha = \frac{\frac{\sqrt{90}}{10}}{\frac{\sqrt{10}}{10}} = \sqrt{9} = \boxed{3}$ ✓

$b^2 = 4^2 + 2^2 \qquad a^2 = 2^2 + 2^2$

$b = \sqrt{20} \qquad a = \sqrt{8}$

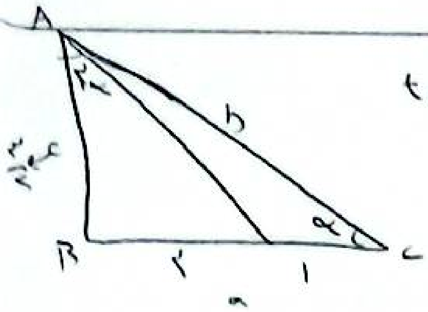
$\sin\alpha = \frac{\sqrt{10}}{10}$

$\Rightarrow \cos\alpha = \frac{\sqrt{90}}{10}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\frac{10}{100} + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{90}{100}$

۳- (۲)



$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{2}{c} = \frac{\frac{2c}{3}}{1 - \frac{c^2}{9}} = \frac{\frac{2c}{3}}{\frac{9 - c^2}{9}} = \frac{6c}{9 - c^2} = \frac{2}{c}$

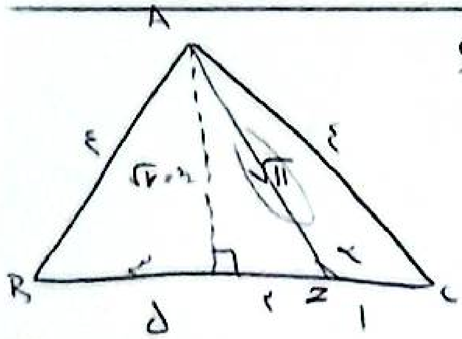
$6c^2 = 18 - 2c^2$

$8c^2 = 18$

$c^2 = \frac{18}{8} = \frac{9}{4} \Rightarrow c = \frac{3}{2}$

$\cot\alpha = \frac{3}{\frac{3}{2}} = \boxed{2}$ ✓

۴- (۲)



$S_{کل} = \frac{\sqrt{7} \times 6}{2} = 3\sqrt{7} = S_{ABZ} + S_{AZC}$ ①

$\frac{\sqrt{7} \times 5}{2}$

$\Rightarrow S_{AZC} = \frac{1}{2}\sqrt{7} = \frac{1}{2} \times \sin\alpha \times \sqrt{11} \times 1$

$\sqrt{7} = \sin\alpha \times \sqrt{11} \Rightarrow \sin\alpha = \sqrt{\frac{7}{11}}$

$\tan(180^\circ - \alpha) = -\tan\alpha$ ②

$\frac{\sqrt{7}}{2} = -\tan\alpha$

$\tan\alpha = \boxed{-\frac{\sqrt{7}}{2}}$

$z^2 + 9 = 16$

$z^2 = 7 \quad z = \sqrt{7}$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos\alpha = \frac{2}{\sqrt{11}}$

$\frac{7}{11} + \boxed{\frac{4}{11}} = 1$

$\tan\alpha = \frac{\frac{\sqrt{7}}{\sqrt{11}}}{\frac{2}{\sqrt{11}}} = \boxed{-\frac{\sqrt{7}}{2}}$ ✓

چون $90^\circ < \alpha$

۵- (۲)

$2\sin^2 x + \cos^2 x = \frac{3}{2}$

$\sin^2 x + \sin^2 x + \cos^2 x = \frac{3}{2} \Rightarrow \sin^2 x = \frac{1}{2} \Rightarrow \cos^2 x = \frac{1}{2}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{2}}{\frac{1}{2}} = \boxed{1}$ ✓

۶- (۲)

$$\frac{\sin^2\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^2\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{(\sin^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(\cos^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha} =$$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha = 1 - \cos^2\alpha$

$\cos^2\alpha = 1 - \sin^2\alpha$

$$\Rightarrow \frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha}$$

$$\frac{\cos^4\alpha + 2\cos^2\alpha + 1}{1+\cos^2\alpha} - \frac{\sin^4\alpha + 2\sin^2\alpha + 1}{1+\sin^2\alpha} =$$

$$\frac{(\cos^2\alpha+1)^2}{\cos^2\alpha+1} - \frac{(\sin^2\alpha+1)^2}{\sin^2\alpha+1} = \cos^2\alpha + 1 - \sin^2\alpha - 1 = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$$

۷- (۲)

ربع سوم $\Rightarrow \sin\alpha = -\frac{4}{5}$

$\cos\alpha = -\frac{3}{5}$

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right)$$

$\frac{\pi}{2}+\alpha$

$\cos\alpha \times -\sin\alpha \qquad +\cot\alpha = -\frac{3}{5}\times\frac{4}{5} = -\frac{12}{25} + \frac{3}{4} = \frac{75-48}{100} = \frac{27}{100}$

$\boxed{\frac{27}{100}}$

۸- (۲)

$\frac{\pi}{12} = \frac{2}{180} \Rightarrow \frac{2}{180} = \frac{1}{12} \Rightarrow 2 = 15^\circ$

$\sin 2^\circ = \sin(45-43) = \sin 45\cos 43 - \sin 43\cos 45 = -\frac{1}{2}$

$$\left(2\cos 43^\circ + \sqrt{2}\sin 2 - \sqrt{2}\cos 2\right) = \cos(45-43) = \cos 45\cos 43 + \sin 45\sin 43 = \frac{\sqrt{3}+\sqrt{2}}{4}$$

$\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}$ $\quad$ $\frac{\sqrt{2}}{2}\times\frac{1}{2}$

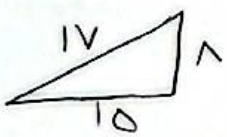
$$\frac{1}{2} + \sqrt{2}(\sin 2 - \cos 2) = \frac{1}{2} - 1 = \boxed{-\frac{1}{2}}$$

$$\frac{\sqrt{3}-\sqrt{2}-(\sqrt{3}+\sqrt{2})}{4} = -\frac{2\sqrt{2}}{4} = -\frac{\sqrt{2}}{2}$$

$\boxed{\frac{1}{4}}$

۹- (۲)

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{3} \qquad \tan\alpha = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{1}{2}}{1-\frac{1}{9}} = \frac{9}{8} = \frac{18}{16} = \frac{9}{8}$$



$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{8(17-15)}{15\times17}}{\frac{-7}{17}} = \frac{16}{15\times-7} = -\frac{16}{105}$$

۱۰- (۲)

$2\sin\alpha < \sin 2\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \qquad 2\sin\alpha(\underbrace{\cos\alpha - 1}_{<0}) > 0$

$0 < \frac{\cot\alpha}{\sin\alpha} \Rightarrow 0 < \frac{\cos\alpha}{\sin^2\alpha} \Rightarrow \cos = +$ $\quad \Rightarrow \sin\alpha < 0 \Rightarrow \sin\alpha = -$

$\sin\alpha = \cos\alpha = + \Rightarrow$ ناحیه ۴