«باسمه تعالی»

نام و نام خانوادگی: علی رضایی

شماره داوطلبی: ۸۰۰۰۰۰۰۰

امضاء: ۲۷

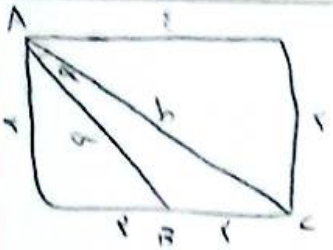
۱-

$S = \frac{1}{2}ab\sin\alpha = \frac{1}{2}\times 4\times\sqrt{5}\times\sin\alpha = 5.0$

$\sqrt{5}\times\sin\alpha = 1.8$

$\Rightarrow \sin\alpha = \frac{\sqrt{5}}{5}$ ∝ { ۱۲۰ ، ۶۰ } $\Rightarrow \frac{120}{60} = 2$

۲-



$S_{ABC} = \frac{1}{2}\times 2\times 2 = 2 = \frac{1}{2}ab\sin\alpha$

$\frac{1}{2}\times\sqrt{20}\times\sqrt{8}\times\sin\alpha = 2$

$\sqrt{160}\times\sin\alpha = 8$

$b^2 = 4^2+2^2$ $\quad a^2 = 2^2+2^2$

$b = \sqrt{20}$ $\quad a = \sqrt{8}$

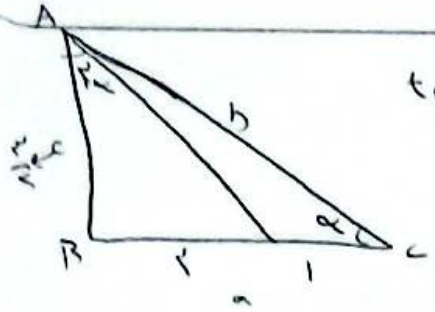
$\sin\alpha = \frac{\sqrt{10}}{10}$

$\Rightarrow \cos\alpha = \frac{\sqrt{90}}{10}$

$\sin^2\alpha+\cos^2\alpha = 1$

$\frac{10}{100}+\cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{90}{100}$

$\cot\alpha = \frac{\sqrt{90}}{\sqrt{10}} = \sqrt{9} = 3$



$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2}{c} = \frac{\frac{2c}{2}}{1-\frac{c^2}{9}} = \frac{\frac{2c}{2}}{\frac{9-c^2}{9}} = \frac{9c}{9-c^2} = \frac{2-c}{c}$

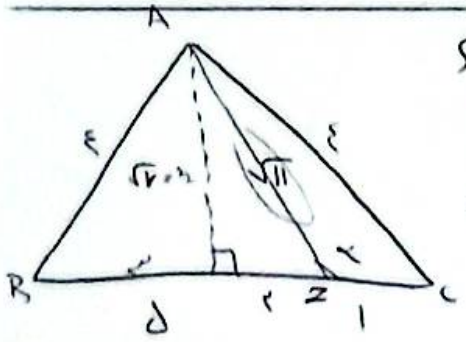
$9c^2 = 18-2c^2$

$8c^2 = 18$

$c^2 = \frac{18}{8} = \frac{9}{4} \Rightarrow c = \frac{3}{2}$

$\cot\alpha = \frac{2}{\frac{3}{2}} = 3$

۳-



$S_{ABC} = \frac{\sqrt{7}\times 4}{2} = 2\sqrt{7} = S_{ABD}+S_{ADC}$ ①

$\frac{\sqrt{7}\times 3}{2}$.

$\Rightarrow S_{ADC} = \frac{1}{2}\sqrt{7} = \frac{1}{2}\times\sin\alpha\times\sqrt{11}\times 1$

$\sqrt{7} = \sin\alpha\times\sqrt{11} \Rightarrow \sin\alpha = \sqrt{\frac{7}{11}}$

$2^2+9 = 14$

$2^2 = 7 \Rightarrow 2 = \sqrt{7}$

$\sin^2\alpha+\cos^2\alpha = 1 \Rightarrow \cos\alpha = \frac{2}{\sqrt{11}}$

$\frac{7}{11}+\left[\frac{2}{\sqrt{11}}\right] = 1$

$\tan(180-\alpha) = -\tan\alpha$ ②

$\frac{\sqrt{7}}{2} = -\tan\alpha$

$\tan\alpha = -\frac{\sqrt{7}}{2}$

$\tan\alpha = \frac{\frac{\sqrt{7}}{\sqrt{11}}}{\frac{2}{\sqrt{11}}} = \frac{\sqrt{7}}{2}$

چون $90 < \alpha < 180$

۵-

$2\sin^2\alpha+\cos^2\alpha = \frac{5}{2}$

$\sin^2\alpha+\sin^2\alpha+\cos^2\alpha = \frac{5}{2}$

$\frac{1}{2}+\cos^2\alpha = \frac{5}{2} \Rightarrow \sin^2\alpha = \frac{1}{2} \Rightarrow \cos^2\alpha = \frac{1}{2}$

$\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{1}$

۶-

$$\frac{\sin^2\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^2\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{(\sin^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(\cos^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha} =$$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha = 1 - \cos^2\alpha$

$\cos^2\alpha = 1 - \sin^2\alpha$

$$\Rightarrow \frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha}$$

$$\frac{\cos^4\alpha + 2\cos^2\alpha + 1}{1+\cos^2\alpha} - \frac{\sin^4\alpha + 2\sin^2\alpha + 1}{1+\sin^2\alpha} =$$

$$\frac{(\cos^2\alpha+1)^2}{\cos^2\alpha+1} - \frac{(\sin^2\alpha+1)^2}{\sin^2\alpha+1} = \cos^2\alpha + 1 - \sin^2\alpha - 1 = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$$

۷-

ربع سوم $\Rightarrow \sin\alpha = -\frac{4}{5}$

$\cos\alpha = -\frac{3}{5}$

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right)$$

$\frac{3\pi}{2}+\alpha$

$\cos\alpha \times -\sin\alpha \quad + \cot\alpha = -\frac{3}{5}\times\frac{4}{5} = -\frac{12}{25} + \frac{3}{4} = \frac{75-48}{100} = \frac{27}{100}$

$\boxed{\frac{27}{100}}$

۸-

$\frac{\pi}{12} = \frac{2}{180} \Rightarrow \frac{2}{180} = \frac{1}{12} \Rightarrow 2 = 15^\circ$

$\sin 2^\circ = \sin(45-43) = \sin 45\cos 43 - \sin 43\cos 45 = -$

$\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} - \frac{1}{2}\times\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\left(2\cos 2^\circ + \sqrt{2}\sin 2 - \sqrt{2}\cos 2\right) = \cos(45-43) = \cos 45\cos 43 + \sin 45\sin 43 = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} \quad \frac{\sqrt{2}}{2}\times\frac{1}{2}$

$\frac{1}{2} + \sqrt{2}(\sin 2 - \cos 2) = \frac{1}{2} - 1 = \boxed{-\frac{1}{2}}$

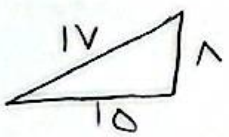
$\frac{\sqrt{6}-\sqrt{2}-(\sqrt{6}+\sqrt{2})}{4} = -\frac{2\sqrt{2}}{4} = -\frac{\sqrt{2}}{2}$

$\boxed{\frac{1}{4}}$

۹-

$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{3}$

$$\tan\alpha = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)} = \frac{\frac{1}{2}}{1-\frac{1}{9}} = \frac{9}{8} = \frac{18}{16} = \frac{8}{15}$$



$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{8(17-15)}{15\times17}}{\frac{-7}{17}} = \frac{16}{15\times-7} = -\frac{16}{105}$$

۱۰-

$2\sin\alpha < \sin 2\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \quad 2\sin\alpha(\underbrace{\cos\alpha - 1}_{<0}) > 0$

$0 < \frac{\cot\alpha}{\sin\alpha} \Rightarrow 0 < \frac{\cos\alpha}{\sin^2\alpha} \Rightarrow \cos = +$ $\Rightarrow \sin\alpha < 0 \Rightarrow \sin\alpha = -$

$\sin\alpha = \cos\alpha = + \Rightarrow$ ناحیه ۴