۱۳٫۷۵

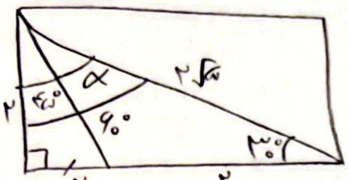
«امیرعلی وحید» «گروه A»

۱- (۲)

$S=\frac{1}{2}ab\sin\alpha \Rightarrow \frac{1}{2}\times\sqrt{3}\times 6\times\sin\alpha=4.5 \rightsquigarrow \sin\alpha=\frac{4.5}{3\sqrt{3}}=\frac{15}{10\sqrt{3}}=\frac{3}{2\sqrt{3}} \rightsquigarrow \frac{3}{2}\times\frac{1}{\sqrt{3}}=\frac{3\sqrt{3}}{6}=\frac{\sqrt{3}}{2}$

120° یا 60° ← $\frac{120}{60}=2$ ✓

۲- ۰



$90=45+\alpha$

$\alpha=15$

$\sin(45-30)=\sin45\cos30-\sin30\cos45=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}=\frac{\sqrt{2}-\sqrt{6}}{4}=\sin15^\circ$

$1+\cot^2 15=\frac{1}{\sin^2 15^\circ}$, $\sin^2 15=\frac{8-4\sqrt{3}}{16}$

$\cot15=\frac{16}{8-4\sqrt{3}}-1=\frac{16-8+4\sqrt{3}}{8-4\sqrt{3}}=\sqrt{\frac{8+4\sqrt{3}}{8-4\sqrt{3}}}=\sqrt{\frac{(\sqrt{2}+\sqrt{6})^2}{(\sqrt{2}-\sqrt{6})^2}}$

$\cot15^\circ=\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}-\sqrt{6}}$

۳- (۰٫۵)

$\cot\alpha=\frac{3}{P}$ ✓

$\cot2\alpha=\frac{P}{2}$

$3\alpha+B=90^\circ$

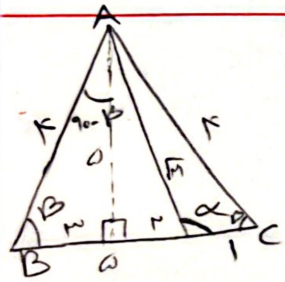
$\cot(2\alpha+B)=\frac{P}{3}$

$\alpha+2\alpha+B=90^\circ$

$90-2\alpha=\alpha+B$

$90-\alpha+\alpha=90^\circ$

۴- ۰



$180-2B=\hat{A}$

$90-B=\frac{\hat{A}}{2}$

۵- (۲)

$\sin x^2+\sin^2x+\cos^2x=\frac{4}{3} \rightsquigarrow \sin^2x=\frac{1}{3} \rightsquigarrow 1+\cot^2x=\frac{1}{\frac{1}{3}} \rightsquigarrow \cot^2x=2$

$\tan^2x=\frac{1}{2}$ ✓

۶- (۱٫۵)

$\sin^2a=x$, $\cos^2a=z$

$\frac{x^2+4z}{1+z}-\frac{z^2+4x}{1+x}=\frac{x^2+4z+x^3+4xz-z^2-4x-z^3-4xz}{1+x+z+zx}=\frac{x^2-z^2+x^3-z^3+4z-4x}{2+xz}$

$=\frac{x-z(1+1-xz-4)}{2+xz}=-z+x$

$\frac{(z-x)(-2-xz)}{2+xz}$ ← $\sin^2a-\cos^2a$

۷- (۲)

$-\cos\alpha\times\sin\alpha+\cot\alpha=\frac{3}{5}\times\frac{-4}{5}+\frac{3}{4}\Rightarrow\frac{-12}{25}+\frac{3}{4}=\frac{-48+75}{100}=\frac{27}{100}$ ✓

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}$, $\frac{16}{9}$, $\frac{25}{9}$

$\cos^2a=\frac{9}{25}\rightarrow\cos a=-\frac{3}{5}$ (چون در ربع سوم است) $\rightarrow1-\frac{9}{25}=\frac{16}{25}\rightarrow\sin\alpha=-\frac{4}{5}$ (چون در ربع سوم است)

۸- (۲)

$\frac{\pi}{12}=15^\circ\rightsquigarrow\sin(45-30)=\sin45\cos30-\sin30\cos45$

$\sin15=\frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos(45-30)=\cos45\cos30+\sin45\sin30=\frac{\sqrt{6}+\sqrt{2}}{4}$

$3\cos\frac{\pi}{3}+\sqrt{2}\sin\frac{\pi}{12}-\sqrt{2}\cos\frac{\pi}{12}=\frac{3}{2}+\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right)=\frac{3}{2}-1=\frac{1}{2}$ ✓

$\sqrt{6}-\sqrt{2}-\sqrt{6}-\sqrt{2}$

۹- (۲)

$\cot\frac{\alpha}{2} - \tan\frac{\alpha}{2} = 2\cot\alpha \rightsquigarrow \cot\alpha = \frac{15}{8} \rightsquigarrow \tan\alpha = \frac{8}{15} \rightsquigarrow 1+\frac{64}{225} = \frac{1}{\cos^2\alpha} \rightarrow \frac{289}{225} = \cos^2\alpha$

$\frac{17}{4} - \frac{2}{\sin\alpha} \rightarrow \frac{8}{17} = \sin\alpha$

$\cos\alpha = \frac{15}{17}$

$\frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17}\cdot\frac{15}{17}} = \frac{\frac{136-120}{255}}{\frac{-7}{17}} = \frac{\frac{16}{255}}{\frac{-7}{17}} = \frac{16}{-105}$ ✓

۱۰- (۱,۷۵)

$0 < \frac{\cos\alpha}{\sin\alpha} = \frac{\cos}{\sin^2\alpha} \rightarrow 0 < \cos\alpha$! چرا؟

$\sin\alpha < \frac{\sin^2\alpha}{2}$

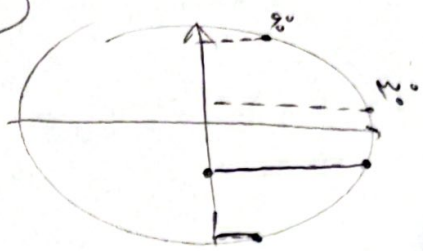
$0 < \frac{\cos\alpha}{\sin^2\alpha} = \frac{\cos\alpha}{\sin^2\alpha\sin\alpha}$

$0 < \sin^2\alpha\sin\alpha$

$\rightarrow \sin\alpha > 0$

$\sin^2\alpha > 0$

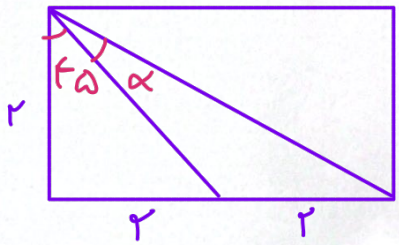
ناصفی



۲-

$\tan(\alpha + \frac{\pi}{4}) = \frac{\tan\alpha + 1}{1 - \tan\alpha} = 2 \rightarrow \tan\alpha + 1 = 2 - 2\tan\alpha$

$3\tan\alpha = 1 \rightarrow \tan\alpha = \frac{1}{3} \rightarrow \cot\alpha = 3$



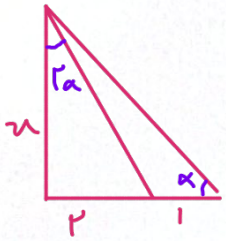
۳-

$\tan 2\alpha = \frac{2}{n}$

$\tan\alpha = \frac{n}{3}$

$\frac{2}{n} = \frac{2(\frac{n}{3})}{1 - \frac{n^2}{9}} \rightarrow n = \frac{3}{2} \rightarrow \tan\alpha = \frac{1}{2}$

$\cot\alpha = 2$



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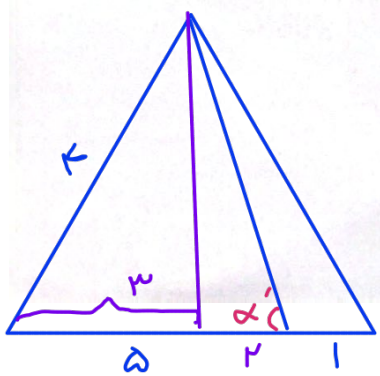

$h = \sqrt{14-9} = \sqrt{5}$

$\tan\alpha' = \frac{\sqrt{5}}{2}$

$\tan\alpha = -\tan\alpha' = -\frac{\sqrt{5}}{2}$

$$\frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{1+(1-\sin^2\alpha)} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{1+(1-\cos^2\alpha)} =$$

$$\frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha} - \frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha} = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$$