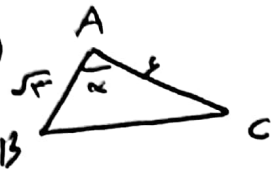
پاسخ جمعه‌ای‌ها / یازدهمی پسر A

۱)

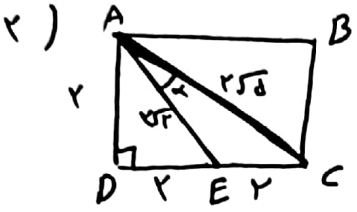


$S=\frac{1}{2}AB\times AC\times\sin\alpha=\frac{1}{2}\times2\sqrt{3}\times4\times\sin\alpha=4\sqrt{3}\sin\alpha$

$\Rightarrow \sin\alpha=\frac{11}{2\sqrt{3}}=\frac{\sqrt{3}}{2}\Rightarrow\begin{cases}\alpha=60^\circ\\ \alpha=120^\circ\end{cases}$ $\frac{120^\circ}{60^\circ}=2$

---

۲)



$\overset{\text{فیثاغورث}}{\underset{ADE}{\Rightarrow}}$ $AE^2=AD^2+DE^2\Rightarrow AE^2=4+4=8\Rightarrow AE=2\sqrt{2}$

$\overset{\text{فیثاغورث}}{\underset{ADC}{\Rightarrow}}$ $AC^2=AD^2+DC^2\Rightarrow AC^2=4+16=20\Rightarrow AC=2\sqrt{5}$

$\overset{\text{قضیه کسینوس‌ها}}{\underset{EAC}{\Rightarrow}}$ $EC^2=AE^2+AC^2-2AE\times AC\times\cos\alpha\Rightarrow 4=8+20-2\times2\sqrt{2}\times2\sqrt{5}\times\cos\alpha$

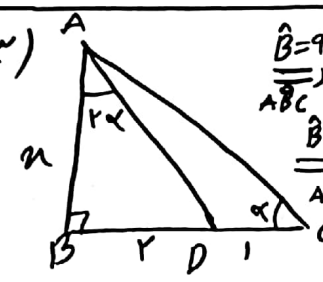
$\Rightarrow 8\sqrt{10}\cos\alpha=24\Rightarrow \boxed{\cos\alpha=\frac{3}{\sqrt{10}}}$ (I) $\sin^2\alpha+\cos^2\alpha=1\overset{(I)}{\Rightarrow}\sin^2\alpha+\frac{9}{10}=1$

$\Rightarrow \sin^2\alpha=\frac{1}{10}\Rightarrow \sin\alpha=\overset{\text{ج ق ا}}{\overset{\alpha<180^\circ}{\oplus}}\frac{1}{\sqrt{10}}\Rightarrow \boxed{\sin\alpha=\frac{1}{\sqrt{10}}}$ (II)

$\cot\alpha=\frac{\cos\alpha}{\sin\alpha}\overset{(I),(II)}{=}\frac{3\sqrt{10}}{\sqrt{10}}=3$

---

۳)



$\overset{\hat B=90^\circ}{\underset{ABC}{\Rightarrow}}$ $\cot\alpha=\frac{BC}{AB}=\frac{3}{n}\overset{(I)}{\Rightarrow}\tan\alpha=\frac{1}{\cot\alpha}=\frac{n}{3}$ (II)

$\overset{\hat B=90^\circ}{\underset{ABD}{\Rightarrow}}$ $\cot2\alpha=\frac{AB}{BD}=\frac{n}{2}\Rightarrow\tan2\alpha=\frac{1}{\cot2\alpha}=\frac{2}{n}$ (III)

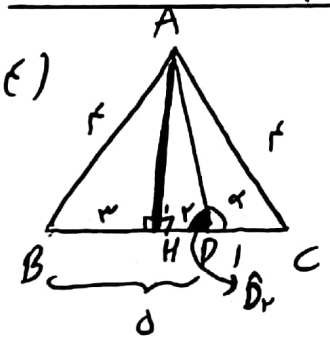
$\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}\overset{(II)}{=}\frac{2(\frac{n}{3})}{1-(\frac{n}{3})^2}=\frac{\frac{2n}{3}}{1-\frac{n^2}{9}}=\frac{\frac{2}{3}n}{\frac{9-n^2}{9}}\overset{III}{=}\frac{2}{n}$

$\Rightarrow\frac{6n}{9-n^2}=\frac{2}{n}\Rightarrow 2n^2=9-n^2\Rightarrow 3n^2=9\Rightarrow n^2=3\Rightarrow n=\overset{\text{ج ق ا}}{\overset{n>0}{\oplus}}\sqrt{3}\Rightarrow n=\sqrt{3}$

$\overset{I}{\Rightarrow}\cot\alpha=\frac{3}{\sqrt{3}}=\sqrt{3}$

---

۴)



$\overset{\text{متساوی‌الساقین}}{\underset{ABC}{\Rightarrow}}$ $AH\rightarrow\begin{cases}\text{ارتفاع و میانه و}\\ \text{BC نیمساز است.}\end{cases}\Rightarrow BH=CH$ (I)

$\left.\begin{array}{l}BC=BD+DC=3+1=4\\ BC=BH+CH\overset{(I)}{=}2CH\end{array}\right\}2CH=4\Rightarrow CH=2$

$CH=CD+DH\Rightarrow DH=1$

$\overset{\text{فیثاغورث}}{\underset{AHB}{\Rightarrow}}$ $AB^2=BH^2+AH^2\Rightarrow 16=9+AH^2\Rightarrow AH^2=7\Rightarrow AH=\sqrt{7}$

$\overset{\hat H=90^\circ}{\underset{AHD}{\Rightarrow}}$ $\tan\hat D_2=\frac{AH}{HD}=\frac{\sqrt{7}}{2}$

$\overset{(II)}{\Rightarrow}\tan\alpha=\boxed{\frac{-\sqrt{7}}{2}}$

$\hat D_2=180^\circ-\alpha\Rightarrow\tan\hat D_2=-\tan\alpha$ (II)

---

۵) $2\sin^2x+\cos^2x=\sin^2x+(\sin^2x+\cos^2x)=\sin^2x+1=\frac{4}{3}\Rightarrow\boxed{\sin^2x=\frac{1}{3}}$ (I)

$\Rightarrow 1-\sin^2x=\boxed{\frac{2}{3}=\cos^2x}$ (II)

$\tan^2x=\frac{\sin^2x}{\cos^2x}\overset{(I),(II)}{=}\frac{\frac{1}{3}}{\frac{2}{3}}=\boxed{\frac{1}{2}}$

۶) $\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}=\frac{(\sin^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha}=\frac{(1-\cos^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha}$

$=\frac{\cos^4\alpha-2\cos^2\alpha+1+4\cos^2\alpha}{1+\cos^2\alpha}=\frac{\cos^4\alpha+2\cos^2\alpha+1}{\cos^2\alpha+1}=\frac{(\cos^2\alpha+1)^2}{\cos^2\alpha+1}=\cos^2\alpha+1$ (I)

$\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{(\cos^2\alpha)^2+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{(1-\sin^2\alpha)^2+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{\sin^4\alpha-2\sin^2\alpha+1+4\sin^2\alpha}{1+\sin^2\alpha}$

$=\frac{\sin^4\alpha+2\sin^2\alpha+1}{\sin^2\alpha+1}=\frac{(\sin^2\alpha+1)^2}{\sin^2\alpha+1}=\sin^2\alpha+1$ (II)

$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}\overset{(I),(II)}{=}\cos^2\alpha+1-\sin^2\alpha-1=\cos^2\alpha-\sin^2\alpha=\boxed{\cos 2\alpha}$

---

۷) $\underbrace{\sin\left(\frac{9\pi}{2}+\alpha\right)}_{\cos\alpha}\underbrace{\cos\left(\frac{7\pi}{2}-\alpha\right)}_{-\sin\alpha}-\underbrace{\tan\left(\alpha-\frac{3\pi}{2}\right)}_{-\cot\alpha}=-\sin\alpha\cos\alpha+\cot\alpha$

$\overset{(I),(II)}{=}-\frac{12}{25}+\frac{3}{4}=\frac{-48+75}{100}=\boxed{\frac{27}{100}}$

$\cot\alpha=\frac{1}{\tan\alpha}=\frac{1}{\frac{4}{3}}=\frac{3}{4}$ (I)

$\cot\alpha+\tan\alpha=\frac{1}{\sin\alpha\cos\alpha}\Rightarrow\sin\alpha\cos\alpha=\frac{1}{\tan\alpha+\cot\alpha}=\frac{1}{\frac{3}{4}+\frac{4}{3}}=\frac{1}{\frac{9+16}{12}}=\frac{12}{25}$ (II)

---

۸) $2\cos^2 4\pi+\sqrt{2}\underbrace{(\sin\pi-\cos\pi)}_{2\sin(\pi-\frac{\pi}{4})}=2\cos^2 4\pi+\sin(\pi-\frac{\pi}{4})\overset{\pi=\frac{\pi}{4}}{=}2\cos^2\frac{\pi}{4}+2\sin(\frac{\pi}{4}-\frac{\pi}{2})$

$=2\cos\frac{\pi}{2}+\sin(-\frac{\pi}{4})=(2)(\frac{1}{2})+2(-\frac{1}{2})=\boxed{0}$

---

۹) $\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{1}{15}-\frac{1}{17}}{\frac{1}{17}-\frac{1}{25}}=\frac{\frac{2}{255}}{\frac{8}{425}}=\frac{425}{1020}=\boxed{\frac{5}{12}}$

$\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}=\frac{2(\frac{1}{3})}{1-\frac{1}{9}}=\frac{\frac{2}{3}}{\frac{8}{9}}=\frac{3}{4}\Rightarrow\begin{cases}\sin\alpha=3K\\ \cos\alpha=4K\end{cases}$

$\begin{cases}\sin^2\alpha+\cos^2\alpha=1\\ \Rightarrow 9K^2+16K^2=1\\ \Rightarrow 25K^2=1\Rightarrow K^2=\frac{1}{25}\\ \Rightarrow\boxed{K=\frac{1}{5}}\end{cases}$

$K=\frac{1}{5}\Rightarrow\begin{cases}\sin\alpha=\frac{3}{5}\\ \cos\alpha=\frac{4}{5}\end{cases}$

$\tan\alpha>0\Rightarrow\sin\alpha,\cos\alpha$ هم‌علامت $\Rightarrow\begin{cases}\sin\alpha>0\\ \cos\alpha>0\end{cases}$ $\sin\alpha=2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$\Rightarrow 3K>0\Rightarrow K>0$

---

۱۰) $\frac{\cot\alpha}{\sin\alpha}>0\Rightarrow\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha}>0\Rightarrow\frac{\cos\alpha}{\sin^2\alpha}>0\Rightarrow\boxed{\cos\alpha>0}$ (I)

$\sin 2\alpha<\sin 4\alpha\overset{\times\cos\alpha}{\underset{\cos\alpha>0}{\Rightarrow}}\underbrace{2\sin\alpha\cos\alpha}_{\sin 2\alpha}<\sin 4\alpha\times\cos\alpha\Rightarrow\sin 2\alpha(\underbrace{1-\cos\alpha}_{+})<0$

$\Rightarrow\sin 2\alpha<0\Rightarrow\underbrace{2\sin\alpha\cos\alpha}_{+}<0\Rightarrow\boxed{\sin\alpha<0}$ (II)

$\overset{(I),(II)}{\Rightarrow}$ α در ناحیه چهارم قرار دارد