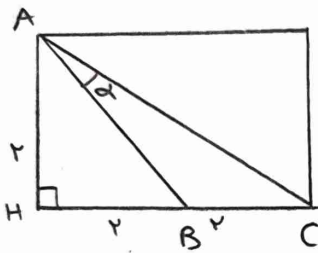




$$S = F \cdot \omega = \frac{1}{2} \sqrt{3} \times 4 \times \sin \alpha \rightarrow \sin \alpha = \frac{\sqrt{3}}{2}$$

$$0 < \alpha < 180^\circ \rightarrow \alpha = 40^\circ, 140^\circ \rightarrow \frac{\alpha_2}{\alpha_1} = 2 \checkmark$$

(۲)



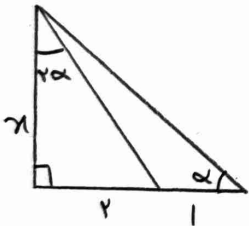
$$S_{ABC} = AH \cdot BC \times \frac{1}{2} = y = \sin \alpha \cdot AB \cdot AC \cdot \frac{1}{2}$$

$$AB = \sqrt{AH^2 + BH^2} = y\sqrt{2}, \quad AC = \sqrt{HC^2 + AH^2} = y\sqrt{2}$$

$$\rightarrow y = \sin \alpha \cdot y\sqrt{2} \cdot y\sqrt{2} \rightarrow \sin \alpha = \frac{\sqrt{10}}{10}$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} = \frac{3\sqrt{10}}{10} \rightarrow \cot \alpha = \frac{\cos \alpha}{\sin \alpha} = 3 \checkmark$$

(۲)

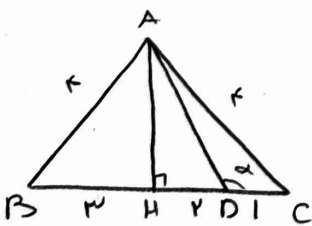


$$\tan \alpha = \frac{x}{y}, \quad \tan 2\alpha = \frac{y}{x}$$

$$\tan 2\alpha = \frac{y \tan \alpha}{1 - \tan^2 \alpha} \rightarrow \frac{y}{x} = \frac{\frac{yx}{y}}{1 - \frac{x^2}{y^2}} \Rightarrow 4x^2 = 1y - yx^2$$

$$\rightarrow x = \frac{y}{4} \rightarrow \tan \alpha = \frac{1}{4} \rightarrow \cot \alpha = 4 \checkmark$$

(۲)



$$S_{ADC} = \frac{1}{2} \times 1 \times AH = \frac{1}{2} \times 1 \times AD \times \sin \alpha$$

$$AH = \sqrt{AB^2 - BH^2} = \sqrt{5}, \quad AD = \sqrt{HD^2 + AH^2} = \sqrt{11}$$

$$\rightarrow \sqrt{11} \sin \alpha = \sqrt{5} \rightarrow \sin \alpha = \frac{\sqrt{5}}{\sqrt{11}}$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha} \rightarrow \cos \alpha = \frac{3}{\sqrt{11}}$$

$$\left. \begin{array}{l} \sin \alpha = \frac{\sqrt{5}}{\sqrt{11}} \\ \cos \alpha = \frac{3}{\sqrt{11}} \end{array} \right\} \xrightarrow{0 < \alpha < 90^\circ} \tan \alpha = \frac{\sqrt{5}}{3} \checkmark$$

(۲)

$$\sin^2 \alpha + \sin^2 \alpha + \cos^2 \alpha = \frac{4}{5} \rightarrow \sin^2 \alpha = \frac{1}{5} \xrightarrow{\sin^2 + \cos^2 = 1} \cos^2 \alpha = \frac{4}{5}$$

$$\rightarrow \tan^2 \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha} = \frac{1}{4} \checkmark$$

(۲)

$$\frac{\sin^2 \alpha + r \cos^2 \alpha}{1 + \cos^2 \alpha} = \frac{(1 - \cos^2 \alpha) + r \cos^2 \alpha}{1 + \cos^2 \alpha} = \frac{\cos^2 \alpha + r \cos^2 \alpha + 1}{1 + \cos^2 \alpha} = \frac{(\cos^2 \alpha + 1) + r \cos^2 \alpha}{1 + \cos^2 \alpha} = \cos^2 \alpha + 1$$

$$9 \quad \frac{\cos^2 \alpha + r \sin^2 \alpha}{1 + \sin^2 \alpha} = \frac{(1 - \sin^2 \alpha) + r \sin^2 \alpha}{1 + \sin^2 \alpha} = \frac{\sin^2 \alpha + r \sin^2 \alpha + 1}{1 + \sin^2 \alpha} = \frac{(1 + \sin^2 \alpha) + r \sin^2 \alpha}{1 + \sin^2 \alpha} = \sin^2 \alpha + 1 \quad (r)$$

$$\rightarrow \frac{\sin^2 \alpha + r \cos^2 \alpha}{1 + \cos^2 \alpha} - \frac{\cos^2 \alpha + r \sin^2 \alpha}{1 + \sin^2 \alpha} = \cos^2 \alpha + 1 - \sin^2 \alpha - 1 = \cos^2 \alpha \quad \checkmark$$

$$\sin\left(\frac{9\pi}{4} + \alpha\right) = \cos \alpha, \quad \cos\left(\frac{5\pi}{4} - \alpha\right) = -\sin \alpha, \quad \tan\left(\alpha - \frac{3\pi}{4}\right) = -\cot \alpha \quad (r)$$

$$\tan \alpha = \frac{r}{k} \rightarrow \begin{array}{c} \text{r} \\ \text{h} \\ \text{k} \end{array} \rightarrow \sin \alpha = \frac{r}{\omega}, \quad \cos \alpha = \frac{k}{\omega}, \quad \cot \alpha = \frac{k}{r}$$

$$\rightarrow \sin\left(\frac{9\pi}{4} + \alpha\right) \cos\left(\frac{5\pi}{4} - \alpha\right) - \tan\left(\alpha - \frac{3\pi}{4}\right) = -\sin \alpha \cos \alpha + \cot \alpha = -\frac{rk}{\omega^2} + \frac{k}{r}$$

$$= \frac{rk}{100} \quad \checkmark$$

$$\sqrt{r} \sin \alpha - \sqrt{r} \cos \alpha = -r (\cos r \alpha \times \cos \alpha - \sin r \alpha \sin \alpha) = -r \cos(r \alpha + \alpha) \quad (r)$$

$$\rightarrow r \cos r \alpha = r \cos\left(\frac{\pi}{r} + \alpha\right) = \cos \frac{\pi}{r} = \frac{1}{r} \quad \checkmark$$

$$\tan^2\left(\frac{\alpha}{4}\right) = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{1}{14} \rightarrow \cos \alpha = \frac{14}{15}, \quad \sin \alpha = \sqrt{1 - \cos^2 \alpha} = \frac{1}{15} \quad (r)$$

$$\rightarrow \tan \alpha = \frac{1}{14}$$

$$\frac{\tan \alpha - \sin \alpha}{\sin \alpha - \cos \alpha} = \frac{\frac{1}{14} - \frac{1}{15}}{\frac{1}{15} - \frac{14}{15}} = \frac{\frac{14}{10 \times 14} - \frac{1}{15}}{\frac{1 - 14}{15}} = \frac{-\frac{14}{100}}{-\frac{13}{15}} = \frac{14}{100} \quad \checkmark$$

$$\frac{\cot \alpha}{\sin \alpha} > 0 \rightarrow \frac{\cos \alpha}{\sin^2 \alpha} > 0 \xrightarrow{\sin^2 \alpha > 0} \cos \alpha > 0 \quad (1)$$

$$\sin^2 \alpha > r \sin \alpha \rightarrow r \sin \alpha (\cos \alpha) > r \sin \alpha \rightarrow r \sin \alpha (\cos \alpha - 1) > 0$$

$$r \sin \alpha (\cos \alpha - 1) > 0 \xrightarrow{1) \cos \alpha > 0} r \sin \alpha (-) > 0 \rightarrow \sin \alpha < 0 \quad (2)$$

$$(1), (2) \rightarrow \text{p, h, n, l} \quad \checkmark$$