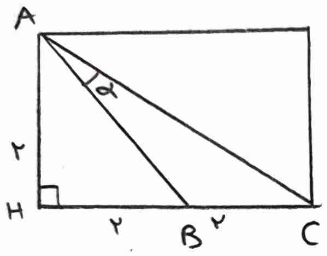
نام و نام خانوادگی: شنتیا پادشاه زبانی　　پاسخنامه تشریحی تکلیف شماره ۲۶　　کلاس یازدهم سپهر

۱

$$S = 4{,}5 = \frac{1}{2}\sqrt{3} \times 6 \times \sin\alpha \rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$$

$$\xrightarrow{0<\alpha<180} \alpha = 60, 120 \rightarrow \frac{\alpha_2}{\alpha_1} = 2$$

۲



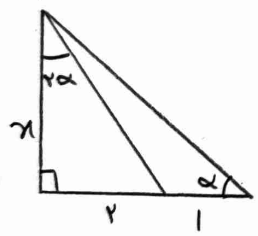
$$S_{ABC} = AH \cdot BC \times \frac{1}{2} = 2 = \sin\alpha \cdot AB \cdot AC \cdot \frac{1}{2}$$

$$AB = \sqrt{AH^2 + BH^2} = 2\sqrt{2}, \quad AC = \sqrt{HC^2 + AH^2} = 2\sqrt{5}$$

$$\rightarrow 4 = \sin\alpha \cdot 2\sqrt{2} \cdot 2\sqrt{5} \rightarrow \sin\alpha = \frac{\sqrt{10}}{10}$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \frac{3\sqrt{10}}{10} \rightarrow \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = 3$$

۳

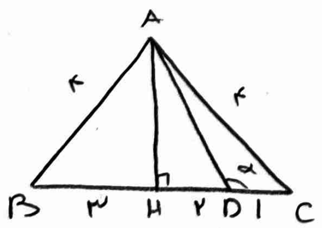


$$\tan\alpha = \frac{x}{3}, \quad \tan 2\alpha = \frac{2}{x}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \rightarrow \frac{2}{x} = \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} \Rightarrow 6x^2 = 18 - 2x^2$$

$$\rightarrow x = \frac{3}{2} \rightarrow \tan\alpha = \frac{1}{2} \rightarrow \cot\alpha = 2$$

۴



$$S_{ADC} = \frac{1}{2} \times 1 \times AH = \frac{1}{2} \times 1 \times AD \times \sin\alpha$$

$$AH = \sqrt{AB^2 - BH^2} = \sqrt{7}, \quad AD = \sqrt{HD^2 + AH^2} = \sqrt{11}$$

$$\left.\begin{array}{l} \rightarrow \sqrt{11}\sin\alpha = \sqrt{7} \rightarrow \sin\alpha = \frac{\sqrt{7}}{\sqrt{11}} \\ \cos\alpha = \sqrt{1-\sin^2\alpha} \rightarrow \cos\alpha = \frac{2}{\sqrt{11}} \end{array}\right\} \xrightarrow{90<\alpha} \tan\alpha = -\frac{\sqrt{7}}{2}$$

۵

$$\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3} \rightarrow \sin^2 x = \frac{1}{3} \xrightarrow{\sin^2 + \cos^2 = 1} \cos^2 x = \frac{2}{3}$$

$$\rightarrow \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1}{2}$$

6

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}=\frac{(1-\cos^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha}=\frac{\cos^4\alpha+2\cos^2\alpha+1}{1+\cos^2\alpha}=\frac{(\cos^2\alpha+1)^2}{\cos^2\alpha+1}=\cos^2\alpha+1$$

$$\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{(1-\sin^2\alpha)^2+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{\sin^4\alpha+2\sin^2\alpha+1}{1+\sin^2\alpha}=\frac{(1+\sin^2\alpha)^2}{1+\sin^2\alpha}=\sin^2\alpha+1$$

$$\rightarrow \frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\cos^2\alpha+1-\sin^2\alpha-1=\cos 2\alpha$$

7

$$\sin\left(\frac{9\pi}{2}+\alpha\right)=\cos\alpha\ ,\ \cos\left(\frac{7\pi}{2}-\alpha\right)=-\sin\alpha\ ,\ \tan\left(\alpha-\frac{3\pi}{2}\right)=-\cot\alpha$$

$$\tan\alpha=\frac{4}{3}\rightarrow \rightarrow \sin\alpha=-\frac{4}{5}\ ,\ \cos\alpha=-\frac{3}{5}\ ,\ \cot\alpha=\frac{3}{4}$$

$$\rightarrow \sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right)=-\sin\alpha\cos\alpha+\cot\alpha=-\frac{12}{25}+\frac{3}{4}$$

$$=\frac{27}{100}$$

8

$$\sqrt{2}\sin x-\sqrt{2}\cos x=-2(\cos 45\times\cos x-\sin 45\sin x)=-2\cos(45+x)$$

$$\rightarrow 3\cos 4x-2\cos\left(\frac{\pi}{4}+x\right)=\cos\frac{\pi}{3}=\frac{1}{2}$$

9

$$\tan^2\left(\frac{\alpha}{2}\right)=\frac{1-\cos\alpha}{1+\cos\alpha}=\frac{1}{16}\rightarrow\cos\alpha=\frac{15}{17}\ ,\ \sin\alpha=\sqrt{1-\cos^2\alpha}=\frac{8}{17}$$

$$\rightarrow\tan\alpha=\frac{8}{15}$$

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{16}{15\times17}}{\frac{-7}{17}}=-\frac{16}{105}$$

10

$$\frac{\cot\alpha}{\sin\alpha}>0\rightarrow\frac{\cos\alpha}{\sin^2\alpha}>0\xrightarrow{\sin^2\alpha>0}\cos\alpha>0 \quad ①$$

$$\sin 2\alpha>2\sin\alpha\rightarrow 2\sin\alpha\cos\alpha>2\sin\alpha\rightarrow 2\sin\alpha\cos\alpha-2\sin\alpha>0$$

$$2\sin\alpha(\cos\alpha-1)>0\xrightarrow{1>\cos\alpha>0}2\sin\alpha(-)>0\rightarrow\sin\alpha<0 \quad ②$$

①,② → ناحیه چهارم