شایان رومی‌زاده

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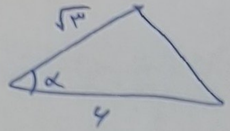
۱ (۲)



$S_\Delta = \frac{1}{2} AB\cos\theta \rightarrow \frac{9}{2} = \frac{1}{2}\sqrt{3}\times 6 \times \sin\alpha \rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$

~~$\rightarrow \alpha = \frac{\pi}{6}$ یا $\frac{\pi}{2}+\frac{\pi}{3}$~~

$\rightarrow \alpha = \frac{\pi}{3}$ یا $\frac{\pi}{2}+\frac{\pi}{6} \rightarrow \alpha = 60,\ \alpha = 120 \rightarrow \frac{120}{60} = \boxed{2}$ ✓

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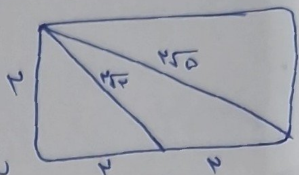
۲ (۲)



$S_{\square} = 2\times 4 = 8$

$S_{\square} = 2 + S_{\triangle} + 4 \rightarrow S_{\triangle} = 2$

$\frac{1}{2}\times 2\sqrt{2}\times 2\sqrt{5}\,\sin\alpha = 2 \rightarrow \sin\alpha = \frac{1}{\sqrt{10}}$ و $\frac{1}{\sin^2\alpha} = 1+\cot^2\alpha \rightarrow \sqrt{10-1} = |\cot\alpha|$

$\rightarrow |\cot\alpha| = 3 \xrightarrow[\cot>0]{\text{زاویه} < 90} \boxed{\cot\alpha = 3}$ ✓

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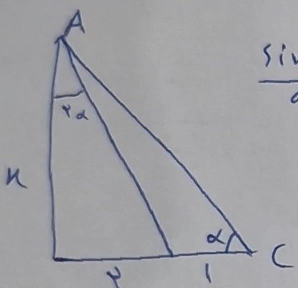

~~$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \rightarrow \frac{\sin 2\alpha}{2} = \frac{\sin 90}{AC} \rightarrow \frac{2\sin\alpha\cos\alpha}{2} = \frac{1}{AC}$~~

~~$\cot\alpha = \frac{3}{AC} = \frac{\cos\alpha}{\sin\alpha} \rightarrow \frac{\cos\alpha}{3\sin\alpha} = \frac{1}{AC}$~~

~~$\frac{1}{AC} = \frac{2\sin\alpha\cos\alpha}{2} = \frac{\cos\alpha}{3\sin\alpha} \rightarrow (\sin\alpha)^2 = \frac{1}{3} \rightarrow |\sin\alpha| = \frac{\sqrt{3}}{3} \xrightarrow{\alpha<\frac{\pi}{2}} \sin\alpha = \frac{\sqrt{3}}{3}$~~

~~$1+\cot^2\alpha = \frac{1}{\sin^2\alpha} \rightarrow \cot^2\alpha = 2 \xrightarrow{\alpha<\frac{\pi}{2}} \boxed{\cot\alpha = \sqrt{2}}$~~

راه ۲

~~$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow \cos^2\alpha = \frac{2}{3}$~~

~~$\rightarrow \cot\alpha = \sqrt{\frac{\cos^2\alpha}{\sin^2\alpha}} = \boxed{\sqrt{2}}$~~

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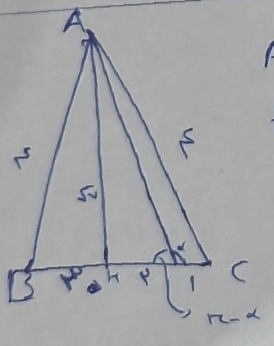
۴ (۲)



$AH^2 = AB^2 - BH^2 \rightarrow AH = \sqrt{7}$

$\tan(\pi-\alpha) = \frac{\sqrt{7}}{2} \rightarrow -\tan\alpha = \frac{\sqrt{7}}{2} \rightarrow \boxed{\tan\alpha = \frac{-\sqrt{7}}{2}}$ ✓

(۲) ۳

$\tan 2\alpha = \frac{3}{x}$ ، $\tan\alpha = \frac{x}{3}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \rightarrow \frac{3}{x} = \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} \rightarrow \frac{2x^2}{3} = 3 - \frac{3x^2}{9}$

$6x^2 = 18 - 3x^2 \rightarrow x = \sqrt{2} \rightarrow \boxed{\cot\alpha = \frac{3}{\sqrt{2}}}$ ✓

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۵ (۲)

$2\sin^2x + \cos^2x = \sin^2x + 1 = \frac{4}{3} \rightarrow |\sin x| = \sqrt{\frac{1}{3}} \rightarrow \sin^2x = \frac{1}{3}$

$\sin^2x + \cos^2x = 1 \rightarrow \frac{1}{3} + \cos^2x = 1 \rightarrow \cos^2x = \frac{2}{3}$

$\tan^2x = \frac{\sin^2x}{\cos^2x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$ ✓

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^4\alpha - 4\sin^2\alpha + 4}{2-\sin^2\alpha} - \frac{\cos^4\alpha - 4\cos^2\alpha + 4}{2-\cos^2\alpha}$$

۶ (۲)

$$\begin{matrix}\sin^2\alpha = t \\ \cos^2\alpha = m\end{matrix} \longrightarrow \frac{t^2-4t+4}{2-t} - \frac{m^2-4m+4}{2-m} = 2-t-(2-m) = -\sin^2\alpha + \cos^2\alpha$$

$$= \boxed{1-2\sin^2\alpha} = \boxed{\cos 2\alpha} \checkmark$$

$$\sin\left(\frac{9\pi}{2}+\alpha\right) = \sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha \qquad \cos\left(\frac{7\pi}{2}-\alpha\right) = \cos\left(\frac{3\pi}{2}-\alpha\right) = -\sin\alpha$$

$$-\tan\left(\alpha-\frac{3\pi}{2}\right) = \tan\left(\frac{3\pi}{2}-\alpha\right) = \cot\alpha$$

۷ (۲)

$$A = (\cos\alpha)(-\sin\alpha) + (\cot\alpha) = -\frac{12}{25} + \frac{3}{4} = \frac{75-48}{100} = \boxed{0.27} \checkmark$$

$$\cos^2\alpha = \frac{1}{\tan^2\alpha+1} = \frac{9}{25} \rightarrow \cos\alpha = \frac{3}{5} \Rightarrow \sin^2\alpha = \frac{16}{25} \qquad \cot\alpha = \frac{1}{\tan\alpha} = \frac{3}{4}$$

$$\left(3\cos^2\frac{\pi}{12} + \sqrt{2}\left(\sin\frac{\pi}{12} - \cos\frac{5\pi}{12}\right)\right) = 3\left(\frac{1}{2}\right) + \sqrt{2}\left(\frac{\sqrt{2-\sqrt{3}}}{2} - \frac{\sqrt{2+\sqrt{3}}}{2}\right) = \frac{3}{2} + \frac{\sqrt{2}}{2}\left(\sqrt{2-\sqrt{3}} - \sqrt{2+\sqrt{3}}\right) = \boxed{\quad}$$

$A < 0$

$\frac{1}{2}$ (۲) ۸

$$\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4} \rightarrow \cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\sin^2\frac{\pi}{12} = \frac{1-\cos\frac{\pi}{6}}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4} \rightarrow \sin\frac{\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$A^2 = 2-\sqrt{3}+2+\sqrt{3}+2(1) = 2 \qquad A = -\sqrt{2}$$

در صورتی که $\cos\alpha, \sin\alpha \neq 0$

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{8\left(\frac{2}{15\times 17}\right)}{-\frac{7}{17}} = \boxed{-\frac{16}{105}} \checkmark$$

$\sin\alpha$: $\frac{15}{17}$, $-\frac{15}{17}$

$\cos\alpha$: $\frac{8}{17}$, $-\frac{8}{17}$

۹ (۲)

$$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \rightarrow 1+\frac{49}{225} = \frac{289}{225} = \frac{1}{\cos^2\alpha} \rightarrow |\cos\alpha| = \frac{15}{17}$$

$$\sin^2\alpha = 1-\frac{225}{289} = \frac{64}{289} \rightarrow |\sin\alpha| = \frac{8}{17}$$

$$\tan\left(\alpha-\frac{\pi}{4}\right) = \frac{\tan\alpha - \tan\frac{\pi}{4}}{1+\tan\alpha\tan\frac{\pi}{4}} \xrightarrow[\tan\frac{\pi}{4}=1]{\tan\alpha = m} \frac{1}{3} = \frac{m-1}{1+m} \rightarrow \frac{1}{3} = \frac{3m-1}{m+3} \rightarrow m+3 = 12m-4 \rightarrow m = \frac{8}{15}$$

$$2\sin\alpha < \sin^2\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \rightarrow \sin\alpha < \sin\alpha\cos\alpha \xrightarrow{\cos\alpha \leq 1} \boxed{\sin\alpha < 0}$$

$$0 < \frac{\cot\alpha}{\sin\alpha} \Rightarrow 0 < \frac{\cos\alpha}{\sin^2\alpha} \rightarrow \boxed{0 < \cos\alpha}$$

(۲) α در ناحیه‌ی چهارم قرار دارد. ✓ | ۱۰