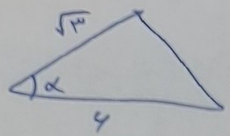
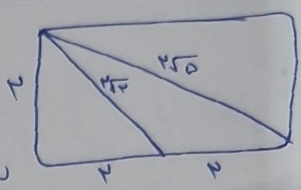
شایان رومی‌زاده



$S_{\Delta} = \frac{1}{2} AB \cos\theta \rightarrow \frac{9}{2} = \frac{1}{2}\sqrt{3} \times 4 \times \sin\alpha \rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$ 1

$\rightarrow \alpha = \frac{\pi}{3}$ یا $\frac{\pi}{2} + \frac{\pi}{6} \rightarrow \begin{matrix} \alpha = 60 \\ \alpha = 120 \end{matrix} \rightarrow \frac{120}{60} = \boxed{2}$

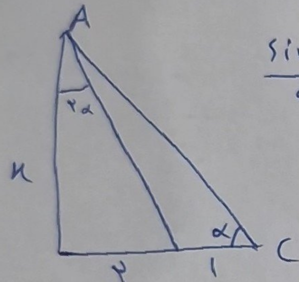


$S_{\square} = 2 \times 4 = 8$

$S_{\square} = 2 + S_{\triangle} + 4 \rightarrow S_{\triangle} = 2$ 2

$\frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{5} \sin\alpha = 2 \rightarrow \sin\alpha = \frac{1}{\sqrt{10}}$ و $\frac{1}{\sin^2\alpha} = 1 + \cot^2\alpha \rightarrow \sqrt{10 - 1} = |\cot\alpha|$

$\rightarrow |\cot\alpha| = 3 \xrightarrow[\cot > 0]{\text{زاویه} < 90} \boxed{\cot\alpha = 3}$



$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \rightarrow \frac{\sin 2\alpha}{2} = \frac{\sin 90}{AC} \rightarrow \frac{2\sin\alpha\cos\alpha}{2} = \frac{1}{AC}$ 3

$\cot\alpha = \frac{3}{AC} = \frac{\cos\alpha}{\sin\alpha} \rightarrow \frac{\cos\alpha}{3\sin\alpha} = \frac{1}{AC}$

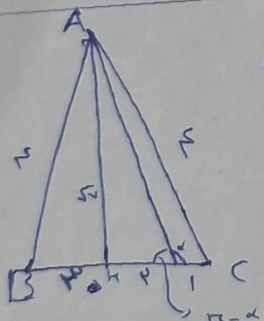
$\frac{1}{AC} = \frac{2\sin\alpha\cos\alpha}{2} = \frac{\cos\alpha}{3\sin\alpha} \rightarrow (\sin\alpha)^2 = \frac{1}{3} \rightarrow |\sin\alpha| = \frac{\sqrt{3}}{3} \xrightarrow{\alpha < \frac{\pi}{2}} \sin\alpha = \frac{\sqrt{3}}{3}$

$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow \cos^2\alpha = \frac{2}{3}$

$\sin^2\alpha = \frac{1}{3}$

$1 + \cot^2\alpha = \frac{1}{\sin^2\alpha} \rightarrow \cot^2\alpha = 2 \xrightarrow{\alpha < \frac{\pi}{2}} \boxed{\cot\alpha = \sqrt{2}}$

$\rightarrow \cot^2\alpha = \sqrt{\frac{\cos^2\alpha}{\sin^2\alpha}} = \boxed{\sqrt{2}}$



$AH^2 = AB^2 - BH^2 \rightarrow AH = \sqrt{7}$

$\tan(\pi - \alpha) = \frac{\sqrt{7}}{2} \rightarrow -\tan\alpha = \frac{\sqrt{7}}{2} \rightarrow \boxed{\tan\alpha = \frac{-\sqrt{7}}{2}}$ 4

$\tan 2\alpha = \frac{2}{x} \quad \tan\alpha = \frac{x}{3}$

-3

$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} \rightarrow \frac{2}{x} = \frac{\frac{2x}{3}}{1 - \frac{x^2}{9}} \rightarrow \frac{2x^2}{3} = 2 - \frac{2x^2}{9}$

$6x^2 = 18 - 2x^2 \rightarrow x = \frac{3}{2} \rightarrow \boxed{\cot\alpha = 2} = \frac{3}{\frac{3}{2}}$

$2\sin^2 x + \cos^2 x = \sin^2 x + 1 = \frac{4}{3} \rightarrow |\sin x| = \sqrt{\frac{1}{3}} \rightarrow \sin^2 x = \frac{1}{3}$

$\sin^2 x + \cos^2 x = 1 \rightarrow \frac{1}{3} + \cos^2 x = 1 \rightarrow \cos^2 x = \frac{2}{3}$ 5

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^4\alpha-4\sin^2\alpha+4}{2-\sin^2\alpha} - \frac{\cos^4\alpha-4\cos^2\alpha+4}{2-\cos^2\alpha}$$ ۶

$$\xrightarrow[\cos^2\alpha=m]{\sin^2\alpha=t} \frac{t^2-4t+4}{2-t} - \frac{m^2-4m+4}{2-m} = 2-t-(2-m) = -\sin^2\alpha+\cos^2\alpha$$

$$= \boxed{1-2\sin^2\alpha} = \boxed{\cos 2\alpha}$$

$\sin(\frac{9\pi}{2}+\alpha) = \sin(\frac{\pi}{2}+\alpha) = \cos\alpha$ | $\cos(\frac{7\pi}{2}-\alpha) = \cos(\frac{3\pi}{2}-\alpha) = -\sin\alpha$

$-\tan(\alpha-\frac{3\pi}{2}) = \tan(\frac{3\pi}{2}-\alpha) = \cot\alpha$ ۷

$A = (\cos\alpha)(-\sin\alpha)+(\cot\alpha) = -\frac{12}{25}+\frac{3}{4} = \frac{75-48}{100} = \boxed{0.27}$

$\cos^2\alpha = \frac{1}{\tan^2\alpha+1} = \frac{9}{25} \to \cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5}$ $\cot\alpha = \frac{1}{\tan\alpha} = \frac{3}{4}$

$\left(3\cos^4\frac{\pi}{12} + \sqrt{2}(\sin\frac{\pi}{12}-\cos\frac{\pi}{12})\right) = 3(\frac{1}{2}) + \sqrt{2}\left(\overbrace{\frac{\sqrt{2-\sqrt{3}}}{2} - \frac{\sqrt{2+\sqrt{3}}}{2}}^{A}\right) = \frac{3}{2} + \frac{\sqrt{2}}{2}\left(\overbrace{\sqrt{2-\sqrt{3}}-\sqrt{2+\sqrt{3}}}^{A<0}\right) = \boxed{\frac{1}{2}}$ ۸

$\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4} \to \cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2}$

$\sin^2\frac{\pi}{12} = \frac{1-\cos\frac{\pi}{6}}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2} = \frac{2-\sqrt{3}}{4} \to \sin\frac{\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2}$ $A^2 = 2-\sqrt{3}+2+\sqrt{3}-2(1) = 2$, $A = -\sqrt{2}$

در صورتی که $0 > \cos\alpha , \sin\alpha$

$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha} = \frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \frac{8(\frac{2}{15\times17})}{\frac{-7}{17}} = \boxed{\frac{-16}{105}}$ $\sin\alpha < \frac{8}{17}, -\frac{8}{17}$ ۹

$\cos\alpha < \frac{15}{17}, -\frac{15}{17}$

$(1+\tan^2\alpha) = \frac{1}{\cos^2\alpha} \to 1+\frac{64}{225} = \frac{289}{225} = \frac{1}{\cos^2\alpha} \to |\cos\alpha| = \frac{15}{17}$; $\sin^2\alpha = 1-\frac{225}{289} = \frac{64}{289} \to |\sin\alpha| = \frac{8}{17}$

$\tan(\alpha-\frac{\alpha}{2}) = \frac{\tan\alpha-\tan\frac{\alpha}{2}}{1+\tan\alpha\tan\frac{\alpha}{2}} \xrightarrow[\tan\frac{\alpha}{2}=\frac{1}{4}]{\tan\alpha=m} \frac{1}{4} = \frac{m-\frac{1}{4}}{1+\frac{m}{4}} \to \frac{1}{4} = \frac{4m-1}{m+4} \to m+4 = 16m-4 \to m = \frac{8}{15}$

$2\sin\alpha < \sin2\alpha \Rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha \to \sin\alpha < \sin\alpha\cos\alpha \xrightarrow[\cos\alpha\le1]{0<\cos\alpha} \boxed{\sin\alpha<0}$

$0 < \frac{\cot\alpha}{\sin\alpha} \Rightarrow 0 < \frac{\cos\alpha}{\sin^2\alpha} \to \boxed{0<\cos\alpha}$ ۱۰

α در ناحیه‌ی چهارم قرار دارد.

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