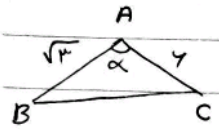
ناهید حیدریان عطار - یازدهم پسر - تالیف

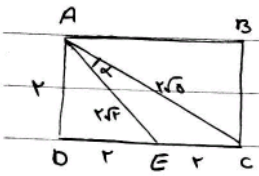
۱- (۲)



$S = \frac{1}{2} AB \times AC \times \sin\alpha = \frac{1}{2} \times \sqrt{3} \times 4 \times \sin\alpha = \frac{4\sqrt{15}}{10}$

$\sin\alpha = \frac{\sqrt{15}}{\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{3}} \rightarrow \alpha = 60^\circ, 120^\circ \quad \frac{120^\circ}{60^\circ} = 2$ ✓

۲- (۲)



$AE^2 = AD^2 + DE^2 \Rightarrow AE^2 = 8 \Rightarrow AE = 2\sqrt{2}$

$AC^2 = AD^2 + DC^2 \Rightarrow AC^2 = 20 \Rightarrow AC = 2\sqrt{5}$

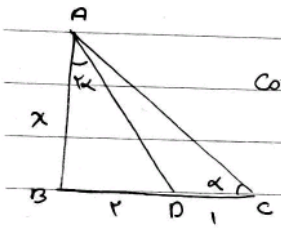
$EC^2 = AE^2 + AC^2 - 2AE \times AC \times \cos\alpha \Rightarrow 16 = 8 + 20 - 2 \times 2\sqrt{2} \times 2\sqrt{5} \times \cos\alpha$

$8\sqrt{10}\cos\alpha = 24 \rightarrow \cos\alpha = \frac{3}{\sqrt{10}} \quad \sin^2\alpha + \cos^2\alpha = 1 \rightarrow \sin^2\alpha + \frac{9}{10} = 1$

$\sin^2\alpha = \frac{1}{10} \quad \sin\alpha = \frac{1}{\sqrt{10}}$ (غ ق ق)

$\cot\alpha = \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{1} = 3$ ✓

۳- (۲)



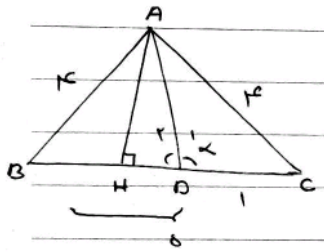
$\cot\alpha = \frac{3}{x} \Rightarrow \tan\alpha = \frac{x}{3} \quad \cot 2\alpha = \frac{x}{2} \Rightarrow \tan 2\alpha = \frac{2}{x}$

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2}{x} \rightarrow \frac{6x}{9-x^2} = \frac{2}{x} \Rightarrow 3x^2 = 9 - x^2 \Rightarrow x^2 = \frac{9}{4}$

$x = \pm\frac{3}{2}$ (غ ق ق)

$\cot\alpha = 2 \times \frac{2}{3} = 4$ ✓

۴- (۲)



$BH = CH, \ BC = 6, \ BC = 2CH \Rightarrow CH = 3$

$CH = CD + DH \Rightarrow DH = 2$

$AB^2 = BH^2 + AH^2 \Rightarrow 16 = 9 + AH^2 \Rightarrow AH^2 = 7 \Rightarrow AH = \sqrt{7}$

$\tan D_2 = \frac{AH}{HD} = \frac{\sqrt{7}}{2} \Rightarrow \tan\alpha = \frac{-\sqrt{7}}{2}$ ✓

$D_2 = 180^\circ - \alpha \Rightarrow \tan D_2 = -\tan\alpha$

۵- (۲)

$2\sin^2\alpha + \cos^2\alpha = \sin^2\alpha + (\sin^2\alpha + \cos^2\alpha) = \sin^2\alpha + 1 = \frac{4}{3} \Rightarrow \sin^2\alpha = \frac{1}{3}$

$1 - \sin^2\alpha = \frac{2}{3} = \cos^2\alpha \quad \tan^2\alpha = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$ ✓

۶-

$$\frac{\sin^4\alpha + 2\cos^2\alpha}{1+\cos^2\alpha} = \frac{(\sin^2\alpha) + 2\cos^2\alpha}{1+\cos^2\alpha} = \frac{(1-\cos^2\alpha) + 2\cos^2\alpha}{1+\cos^2\alpha} = \frac{\cos^4\alpha - 2\cos^2\alpha + 1 + 2\cos^2\alpha}{1+\cos^2\alpha}$$

(۲)

$$= \frac{\cos^4\alpha + 2\cos^2\alpha + 1}{\cos^2\alpha + 1} = \frac{(\cos^2\alpha + 1)^2}{\cos^2\alpha + 1} = \cos^2\alpha + 1$$

$$\frac{\cos^4\alpha + 2\sin^2\alpha}{1+\sin^2\alpha} = \frac{(\cos^2\alpha)^2 + 2\sin^2\alpha}{1+\sin^2\alpha} = \frac{(1-\sin^2\alpha)^2 + 2\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^4\alpha - 2\sin^2\alpha + 1 + 2\sin^2\alpha}{1+\sin^2\alpha}$$

$$= \frac{\sin^4\alpha + 2\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 2\sin^2\alpha}{1+\sin^2\alpha} \rightarrow \cos^2\alpha + 1 - \sin^2\alpha - 1 = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$ ✓

/ /

۷- $\underbrace{\sin(\frac{9\pi}{2}+\alpha)}_{\cos\alpha}\ \underbrace{\cos(\frac{7\pi}{2}-\alpha)}_{-\sin\alpha} - \underbrace{\tan(\alpha-\frac{3\pi}{2})}_{+\cot\alpha} = -\sin\alpha\cos\alpha + \cot\alpha$ (۲)

$\cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$ $\qquad \cot\alpha + \tan\alpha = \frac{1}{\sin\alpha\cos\alpha} \Rightarrow \sin\alpha\cos\alpha = \frac{1}{\tan\alpha+\cot\alpha} = \frac{12}{25}$

$\frac{-12}{25} + \frac{3}{4} = \frac{27}{100}$ ✓

۸- $3\cos 4x + \underbrace{\sqrt{2}(\sin x - \cos x)}_{2\sin(x-\frac{\pi}{4})} = 3\cos 4x + \sin(x-\frac{\pi}{4}) \rightarrow 3\cos\frac{\pi}{3} + 2\sin(\frac{\pi}{12}-\frac{\pi}{4})$ (۲)

$= 3\cos\frac{\pi}{3} + \sin(\frac{-\pi}{6}) = \frac{1}{2}$ ✓

۹- $\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{14}{-105}$ ✓ $\qquad \tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{8}{15}$ → $\sin\alpha = 8k$, $\cos\alpha = 15k$ (۲)

$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow 289k^2 = 1 \Rightarrow k = \pm\frac{1}{17}$ (غ ق ق)

$k = \frac{1}{17} \longrightarrow \sin\alpha = \frac{8}{17}$

$\cos\alpha = \frac{15}{17}$

۱۰- $\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \Rightarrow \frac{\cos\alpha}{\underbrace{\sin^2\alpha}_{+}} > 0 \Rightarrow \underbrace{\cos\alpha > 0}_{(1)}$ (۲)

$2\sin\alpha < \sin 2\alpha \longrightarrow \underbrace{2\sin\alpha\cos\alpha}_{\sin 2\alpha} < \sin 2\alpha \times \cos\alpha \Rightarrow \sin 2\alpha \underbrace{(1-\cos\alpha)}_{+} < 0$

$\sin 2\alpha < 0 \Rightarrow 2\sin\alpha\cos\alpha < 0 \Rightarrow \underbrace{\sin\alpha < 0}_{(2)}$ $\qquad (1),(2) =$ ناحیه چهارم ✓