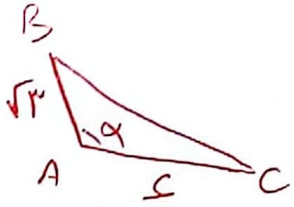
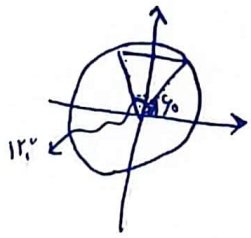
۱۸,۲۵

سوال ۱)



$S_{\Delta} = ۴/۵$



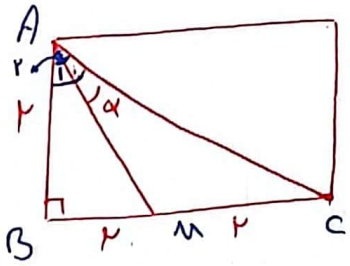
$AB \times AC \times \sin\hat{A} \times \frac{1}{۲} = ۴/۵$ (۲)

$۶ \times \sqrt{۳} \times \sin\hat{A} \times \frac{1}{۲} = ۴/۹ \rightarrow \sin\hat{A} = \frac{\sqrt{۳}}{۲}$

$\alpha_{min} = ۶۰°$

$\alpha_{max} = ۱۲۰° \Rightarrow \frac{\alpha_{max}}{\alpha_{min}} = \frac{۱۲۰}{۶۰} = ۲$ ✓

سوال ۲)

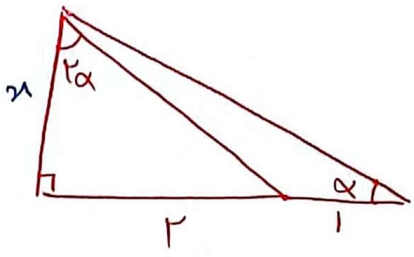


$\cot(A_۲ - A_۱) = \frac{1}{\tan(A_۲ - A_۱)} = \frac{1 + \tan A_۲ \times \tan A_۱}{\tan A_۲ - \tan A_۱}$ (۲)

$\tan\hat{A}_۱ = \frac{BM}{AB} = \frac{۲}{۲} = ۱$ $\quad \tan\hat{A}_۲ = \frac{BC}{AB} = \frac{۴}{۲} = ۲$

$\cot(A_۲ - A_۱) = \frac{۱ + ۲ \times ۱}{۲ - ۱} = \frac{۳}{۱} = ۳$ ✓

سوال ۳)

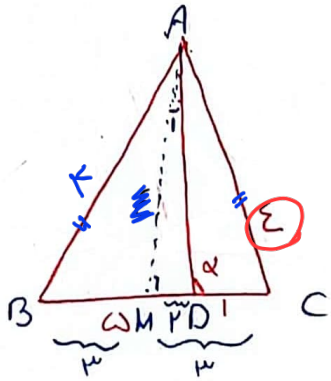


$\tan ۲\alpha = \frac{۲\tan\alpha}{۱ - \tan^۲\alpha}$ $\quad \tan ۲\alpha = \frac{۲}{x}$ $\quad \tan\alpha = \frac{x}{۳}$ (۲)

$\frac{۲}{x} = \frac{۶(\frac{x}{۳})}{۱ - (\frac{x}{۳})^۲}$ $\quad ۳ - \frac{x^۲}{۳} = x^۲$ $\quad \frac{۴}{۳}x^۲ = ۳$ $\quad x^۲ = \frac{۹}{۴}$ $\quad x = \frac{۳}{۲}$

$\Rightarrow \tan\alpha = \frac{\frac{۳}{۲}}{۳} = \frac{۱}{۲} \Rightarrow \cot\alpha = \frac{۱}{\tan\alpha} = \frac{۱}{\frac{۱}{۲}} = ۲$ ✓

سوال ۴)



ABC → متساوی الساقین (۱)

$AH = \sqrt{AC^۲ - HC^۲} = \sqrt{۵^۲ - ۳^۲} = \sqrt{۱۶}$ $\quad AH = \sqrt{۴^۲ - ۳^۲} = \sqrt{۷}$

$\alpha = A_۱ + ۹۰° \Rightarrow \tan(\alpha) = \tan(A_۱ + \frac{\pi}{۲}) = -\cot(A_۱)$

$-\cot A_۱ = -\frac{AH}{HD} = -\frac{\sqrt{۷}}{۲}$ ~~$= -۲$~~

سوال ۵)

$\begin{cases} ۲\sin^۲ x + \cos^۲ x = \frac{۴}{۳} \\ \sin^۲ x + \cos^۲ x = ۱ \end{cases} \xrightarrow{-} \sin^۲ x = \frac{۱}{۳} \Rightarrow \sin^۲ x + \cos^۲ x = ۱ \Rightarrow \cos^۲ x = \frac{۲}{۳}$ (۲)

$\tan^۲ x = \frac{\sin^۲ x}{\cos^۲ x} = \frac{\frac{۱}{۳}}{\frac{۲}{۳}} = \frac{۱}{۲}$ ✓

سوال ۶

$$\frac{\sin^4\alpha+\varepsilon\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+\varepsilon\sin^2\alpha}{1+\sin^2\alpha}=\frac{\sin^4\alpha+\varepsilon(1-\sin^2\alpha)}{1+\cos^2\alpha}-\frac{\cos^4\alpha+\varepsilon(1-\cos^2\alpha)}{1+\sin^2\alpha}$$

$$=\frac{\sin^4\alpha-\varepsilon\sin^2\alpha+\varepsilon}{1+\cos^2\alpha}-\frac{\cos^4\alpha-\varepsilon\cos^2\alpha+\varepsilon}{1+\sin^2\alpha}=\frac{(\sin^2\alpha-1)^2}{1+\cos^2\alpha}-\frac{(\cos^2\alpha-1)^2}{1+\sin^2\alpha}$$

$\cos^2\alpha-\sin^2\alpha=\cos 2\alpha$

$$=\frac{(\sin^2\alpha-1)^2}{2-\sin^2\alpha}-\frac{(\cos^2\alpha-1)^2}{2-\cos^2\alpha}=2-\sin^2\alpha-2+\cos^2\alpha=\cos^2\alpha-\sin^2\alpha=\cos 2\alpha=1$$

سوال ۷

$\tan\alpha=\frac{\varepsilon}{3}\rightarrow$ مجاور α

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{\pi}{2}\right)$$

زاویه دوم منفی $\sin\alpha$

$$(+\cos(\alpha)).(-\sin(\alpha))-(-\cot(\alpha))=-\cos(\alpha)\sin(\alpha)+\cot(\alpha)$$

$$=\frac{1}{\tan\alpha+\cot\alpha}+\cot\alpha=\frac{1}{\frac{\varepsilon}{3}+\frac{3}{\varepsilon}}+\frac{3}{\varepsilon}=\frac{1}{\frac{16+9}{12}}+\frac{3}{\varepsilon}=\frac{12}{25}+\frac{3}{\varepsilon}=\frac{48+75}{100}$$

$$=\frac{123}{100}=1.23$$

سوال ۸

$x=\frac{\pi}{12}$

$\cos 60^\circ$ $\sin 45^\circ$

$$(3\cos 4x+\sqrt{2}\sin x-\sqrt{2}\cos x)\times\frac{1}{2}=\left(\frac{3}{2}\cos 4x+\frac{\sqrt{2}}{2}\sin x-\frac{\sqrt{2}}{2}\cos x\right)$$

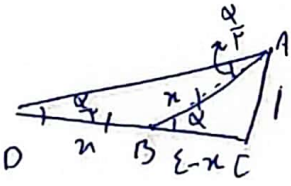
$$=\frac{3}{2}\cos\frac{4\pi}{3}+\sin\left(\frac{\pi}{12}-\frac{\pi}{4}\right)=\frac{3}{2}\cos\frac{\pi}{3}+\sin\left(-\frac{\pi}{6}\right)=\frac{3}{2}\times\frac{1}{2}-\frac{1}{2}=\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$$

$\frac{1}{4}\times 2=\frac{1}{2}$ ✓

سوال ۹

$\tan\left(\frac{\alpha}{2}\right)=\frac{1}{3}$

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}$$

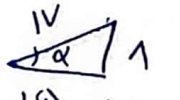


طول وتر ABC: $AB^2=AC^2+BC^2$

$x^2=1+x^2+1-2x$

$2x=1$ $x=\frac{1}{2}$

$$\tan\alpha=\frac{AC}{BC}=\frac{1}{\frac{2\sqrt{2}}{2}}=\frac{2}{\sqrt{8}}$$



$\Rightarrow \sin\alpha=\frac{2}{3}$

$\cos\alpha=\frac{\sqrt{8}}{3}$

$$\frac{\frac{2}{\sqrt{8}}-\frac{2}{3}}{\frac{2}{3}-\frac{\sqrt{8}}{3}}=\frac{\frac{3\times 2-2\times\sqrt{8}}{3\sqrt{8}}}{\frac{3-\sqrt{8}}{3}}=\frac{\frac{6-2\sqrt{8}}{3\sqrt{8}}}{\frac{3-\sqrt{8}}{3}}=\frac{6-2\sqrt{8}}{\sqrt{8}(3-\sqrt{8})}=\frac{2(3-\sqrt{8})}{\sqrt{8}(3-\sqrt{8})}=\frac{2}{\sqrt{8}}$$

✓

سوال ۱۰

$2\sin\alpha<\sin 2\alpha$ $\quad\frac{\cot\alpha}{\sin\alpha}>0$ $\quad 2\sin\alpha<2\sin\alpha\cos\alpha$ $\quad 2\sin\alpha(1-\cos\alpha)<0$

$\Rightarrow -1<\cos\alpha<1$ $\quad 0<-\cos\alpha+1<2\Rightarrow \sin\alpha<0$ I , $\frac{\cos\alpha}{\sin^2\alpha}>0\rightarrow \cos\alpha>0$ II

$\rightarrow$ ناحیه مشترک I و II

✓