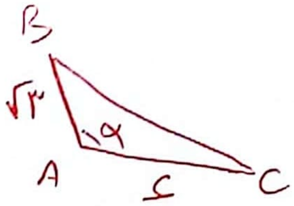
[illegible] ـ [illegible] تالیف ۲۷ کرس دریچه دنیا

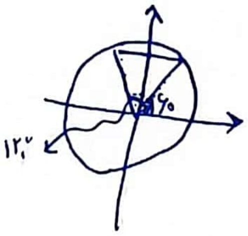
سوال ۱)



$S_{\Delta} = ۴/۵$

$AB \times AC \times \sin\hat{A} \times \frac{1}{۲} = ۴/۵$

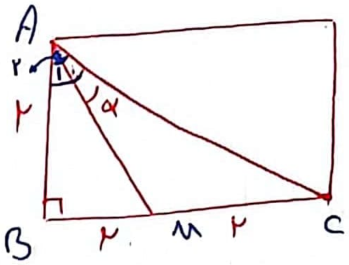
$۶ \times \sqrt{۳} \times \sin\hat{A} \times \frac{1}{۲} = ۴/۹ \longrightarrow \sin\hat{A} = \frac{\sqrt{۳}}{۲}$



$\alpha_{min} = ۶۰°$

$\alpha_{max} = ۱۲۰° \Rightarrow \frac{\alpha_{max}}{\alpha_{min}} = \frac{۱۲۰}{۶۰} = \boxed{۲}$

سوال ۲)

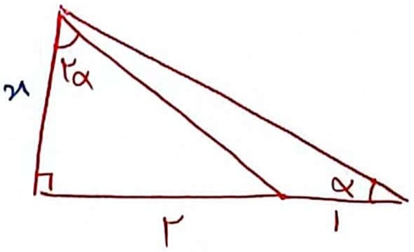


$\cot(A_۲ - A_1) = \frac{1}{\tan(A_۲ - A_1)} = \frac{1 + \tan(A_۲) \times \tan(A_1)}{\tan A_۲ - \tan A_1}$

$\tan\hat{A}_1 = \frac{BM}{AB} = \frac{۲}{۲} = 1 \quad \tan\hat{A}_۲ = \frac{BC}{AB} = \frac{۴}{۲} = ۲$

$\cot(\overset{\alpha}{A_۲ - A_1}) = \frac{1 + ۲ \times 1}{۲ - 1} = \frac{۳}{1} = \boxed{۳}$

سوال ۳)



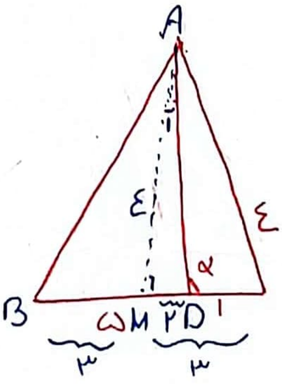
$\tan ۲\alpha = \frac{۲\tan\alpha}{1 - \tan^۲\alpha} \qquad \tan ۲\alpha = \frac{۳}{x} \qquad \tan\alpha = \frac{x}{۳}$

$\frac{۳}{x} = \frac{۲(\frac{x}{۳})}{1 - (\frac{x}{۳})^۲} \qquad ۳ - \frac{x^۲}{۳} = x^۲ \qquad \frac{۴}{۳}x^۲ = ۳ \qquad x^۲ = \frac{۹}{۴}$

$x = \frac{۳}{۲}$

$\Rightarrow \tan\alpha = \frac{\frac{۳}{۲}}{۳} = \frac{1}{۲} \Rightarrow \cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{\frac{1}{۲}} = \boxed{۲}$

سوال ۴)



$ABC \longrightarrow$ متساوی الساقین

$AH = \sqrt{AC^۲ - HC^۲} = \sqrt{۵^۲ - ۳^۲} = ۴$

$\alpha = A_1 + ۹۰° \Rightarrow \tan(\alpha) = \tan(A_1 + \frac{\pi}{۲}) = -\cot(A_1)$

$-\cot A_1 = -\frac{AH}{HD} = -\frac{۴}{۲} = \boxed{-۲}$

سوال ۵)

$\begin{cases} ۲\sin^۲x + \cos^۲x = \frac{۴}{۳} \\ \sin^۲x + \cos^۲x = 1 \end{cases} \xrightarrow{-} \sin^۲x = \frac{1}{۳} \Rightarrow \underset{\frac{1}{۳}}{\sin^۲} + \cos^۲ = 1 \Rightarrow \cos^۲x = \frac{۲}{۳}$

$\tan^۲x = \frac{\sin^۲x}{\cos^۲x} = \frac{\frac{1}{۳}}{\frac{۲}{۳}} = \boxed{\frac{1}{۲}}$

سوال ۶)

$$\frac{\sin^{۴}\alpha+۴\cos^{۲}\alpha}{۱+\cos^{۲}\alpha}-\frac{\cos^{۴}\alpha+۴\sin^{۲}\alpha}{۱+\sin^{۲}\alpha}=\frac{\sin^{۴}\alpha+۴(۱-\sin^{۲}\alpha)}{۱+\cos^{۲}\alpha}-\frac{\cos^{۴}\alpha+۴(۱-\cos^{۲}\alpha)}{۱+\sin^{۲}\alpha}$$

$$=\frac{\sin^{۴}\alpha-۴\sin^{۲}\alpha+۴}{۱+\cos^{۲}\alpha}+\frac{\cos^{۴}\alpha-۴\cos^{۲}\alpha+۴}{۱+\sin^{۲}\alpha}=\frac{(\sin^{۲}-۲)^{۲}}{۱+\cos^{۲}\alpha}+\frac{(\cos^{۲}-۲)^{۲}}{۱+\sin^{۲}\alpha}$$

$$=\frac{(\sin^{۲}_{\alpha}-۲)^{۲}}{۲-\sin^{۲}\alpha}+\frac{(\cos^{۲}_{\alpha}-۲)^{۲}}{۲-\cos^{۲}_{\alpha}}=۲-\sin^{۲}\alpha+۲-\cos^{۲}\alpha=۴-(\underbrace{\sin^{۲}\alpha+\cos^{۲}\alpha}_{۱})=۴-۱=\boxed{۳}$$

سوال ۷)

$\tan\alpha=\frac{۴}{۳}\rightarrow$ α در ربع سوم

$$\sin\left(\frac{۹\pi}{۲}+\alpha\right)\cos\left(\frac{۷\pi}{۲}-\alpha\right)-\tan\left(\alpha-\frac{۳\pi}{۲}\right)$$

$$(-\cos(\alpha)).(-\sin(\alpha))-(-\cot(\alpha))=\cos(\alpha)\sin(\alpha)+\cot(\alpha)$$

$$=\frac{۱}{\tan\alpha+\cot\alpha}+\cot\alpha=\frac{۱}{\frac{۴}{۳}+\frac{۳}{۴}}+\frac{۳}{۴}=\frac{۱}{\frac{۱۶+۹}{۱۲}}+\frac{۳}{۴}=\frac{۱۲}{۲۵}+\frac{۳}{۴}=\frac{۴۸+۷۵}{۱۰۰}$$

$$=\frac{۱۲۳}{۱۰۰}=\boxed{۱/۲۳}$$

سوال ۸)

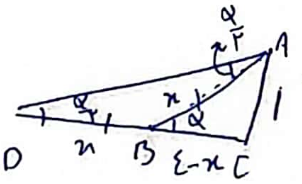
$$\left(۳\cos۴x+\sqrt{۲}\sin x-\sqrt{۲}\cos x\right)\times\frac{۱}{۲}\overset{x=\frac{\pi}{۱۲}}{=}\left(\frac{۳}{۲}\cos۴x+\overset{\cos۴۵^{\circ}}{\frac{\sqrt{۲}}{۲}}\sin x-\overset{\sin۴۵^{\circ}}{\frac{\sqrt{۲}}{۲}}\cos x\right)$$

$$=\frac{۳}{۲}\cos\frac{\pi}{۳}+\sin\left(\frac{\pi}{۱۲}-\frac{\pi}{۴}\right)=\frac{۳}{۲}\cos\frac{\pi}{۳}+\sin\left(-\frac{\pi}{۶}\right)=\frac{۳}{۲}\times\frac{۱}{۲}-\frac{۱}{۲}=\frac{۳}{۴}-\frac{۲}{۴}=\frac{۱}{۴}$$

$$\frac{۱}{۴}\times۲=\boxed{\frac{۱}{۲}}$$

سوال ۹)

$$\tan\left(\frac{\alpha}{۲}\right)=\frac{۱}{۴}\qquad\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}$$



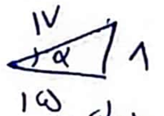
طبق فیثاغورث در $\overset{\triangle}{ABC}$:

$AB^{۲}=AC^{۲}+BC^{۲}$

$x^{۲}=۱+x^{۲}+۱۶-۸x$

$۸x=۱۷\quad x=\frac{۱۷}{۸}$

$$\tan\alpha=\frac{AC}{BC}=\frac{۱}{\frac{۳۲-۱۷}{۸}}=\frac{۸}{۱۵}$$



اعداد فیثاغورثی

$$\Rightarrow\sin\alpha=\frac{۸}{۱۷}\qquad\cos\alpha=\frac{۱۵}{۱۷}$$

$$\frac{\frac{۸}{۱۵}-\frac{۸}{۱۷}}{\frac{۸}{۱۷}-\frac{۱۵}{۱۷}}=\frac{\frac{۱۷\times۸-۸\times۱۵}{۱۷\times۱۵}}{\frac{-۷}{۱۷}}=\frac{\frac{۸\times۲}{\cancel{۱۷}\times۱۵}}{\frac{-۷}{\cancel{۱۷}}}=\frac{۱۶}{-۱۰۵}$$

سوال ۱۰)

$$۲\sin\alpha<\sin۲\alpha\qquad\frac{\cot\,\text{[illegible]}\alpha}{\sin\alpha}>۰\qquad۲\sin\alpha<۲\sin\alpha\times\cos\alpha\qquad۲\sin\alpha(۱-\cos\alpha)<۰$$

$$\Rightarrow-۱<\cos\alpha<۱\qquad۰<-\cos\alpha+۱<۲\Rightarrow\sin\alpha<۰\ \text{I}\ ,\ \frac{\cos\alpha}{\sin\alpha^{۲}}>۰\longrightarrow\cos\alpha>۰\ \text{II}$$

$\overrightarrow{\text{I و II}}$ ربع چهارم