کیان حضری

Date Subject

۱- $\sqrt{۳} \times ۶ \times \frac{۱}{۲} \times \sin\alpha = ۲,۵ \Rightarrow \sin\alpha = \frac{\sqrt{۳}}{۲}$

$\alpha = ۶۰^\circ$ یا $۱۲۰^\circ \rightarrow \frac{۱۲۰}{۶۰} = ۲$ ✓

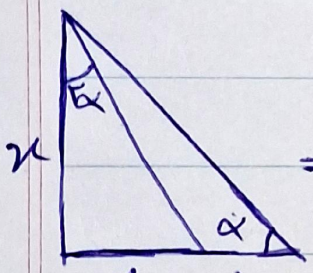
۲- $\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{۱ - \tan(\alpha)\tan(\beta)}$

$\Rightarrow ۲ = \frac{\tan(\alpha)+۱}{۱-\tan(\alpha)} \Rightarrow ۲ - ۲\tan(\alpha) = \tan(\alpha)+۱$

$۱ = ۳\tan\alpha$

$\Rightarrow \tan\alpha = \frac{۱}{۳} \rightarrow \cot\alpha = ۳$ ✓

۳-

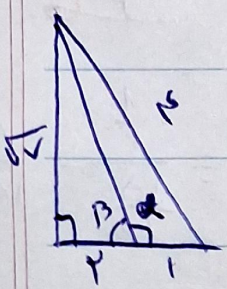


$\tan\alpha = \frac{x}{۳}$ $\quad \tan۲\alpha = \frac{۲}{x} = \frac{\frac{۲x}{۳}}{۱-\frac{x^۲}{۹}} = \frac{۶x}{۹-x^۲}$

$\Rightarrow \frac{۲}{x} = \frac{۶x}{۹-x^۲} \Rightarrow x = ۱,۵ \Rightarrow \tan\alpha = \frac{۲}{۲\times۲}$

$\Rightarrow \cot\alpha = ۲$ ✓

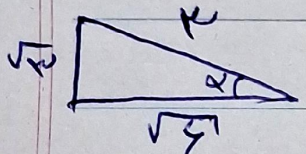
۴-



$\beta + \alpha = \pi \Rightarrow \tan\alpha = -\tan\beta$

$\tan\beta = \frac{\sqrt{۷}}{۲} \Rightarrow \tan\alpha = \frac{-\sqrt{۷}}{۲}$ ✓

۵- $۲\sin^۲x + \cos^۲x = \frac{۴}{۳} \Rightarrow \sin^۲x = \frac{۱}{۳} \Rightarrow \sin x = \frac{\sqrt{۳}}{۳}$



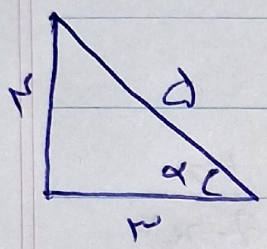
$\Rightarrow \tan^۲x = \left(\frac{\sqrt{۳}}{\sqrt{۶}}\right)^۲ = \frac{۱}{۲}$ ✓

۶- $\frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{1+\sin\alpha}$

$= \frac{(1+\cos^2\alpha)^2}{1+\cos^2\alpha} - \frac{(1+\sin^2\alpha)^2}{1+\sin^2\alpha} = 1+\cos^2\alpha - 1 - \sin^2\alpha = \cos 2\alpha$

۷- $\sin\left(\frac{\pi}{2}+\alpha\right)\cos\left(\frac{3\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right)$

$-\cos(\alpha)\sin(\alpha) + \cot(\alpha)$



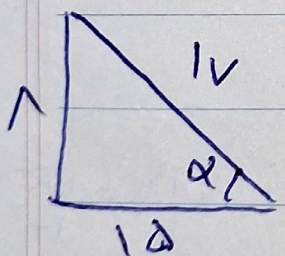
$\Rightarrow -\frac{3}{5}\times\frac{4}{5} + \frac{3}{4} = \frac{-12}{25} + \frac{3}{4} = \frac{27}{100}$

۸- $3\cos^2 x + \sqrt{3}(\sin x - \cos x)$

$3\cos^2 x + \sqrt{3}\left(\sqrt{1-\sin^2 x}\right)$

$x = \frac{\pi}{12} \Rightarrow 3\cos\frac{\pi}{3} + \sqrt{3}\left(\sqrt{1-\sin^2\frac{\pi}{6}}\right) = \frac{3}{2} + \frac{\sqrt{3}}{\sqrt{3}} = \frac{15}{2}$

۹- $\tan\alpha = \frac{2\tan(\frac{\alpha}{2})}{1-\tan(\frac{\alpha}{2})^2} = \frac{\frac{1}{2}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$



$\frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{-17}{105}$

Date Subject

۱۵- ~~$2\sin\alpha < 2\sin\alpha\cos\alpha \Rightarrow 1 < \cos\alpha$~~

۱۰- ~~$2\sin\alpha < 2\sin\alpha\cos\alpha$~~ $\Rightarrow 0 < \sin\alpha\cos\alpha - \sin\alpha$

$\Rightarrow 0 < \sin\alpha \underbrace{(\cos\alpha - 1)}_{(-)} \Rightarrow \sin\alpha < 0$ (I)

$\frac{\cos\alpha}{\sin^2\alpha} > 0 \Rightarrow \cos\alpha > 0$ (II)

$\Rightarrow$ (I), (II) $\Rightarrow \alpha$ در ناحیه‌ی چهارم بوده