سید سپهر میرعبدالباقی یازدهم ۸

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$S=\frac{1}{2}ab\sin\alpha \qquad 4/5=\frac{1}{2}\times 6\times\sqrt{3}\,a\sin\alpha \qquad \sin\alpha=\frac{\sqrt{3}}{2}$

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$\sin(120^\circ)=\frac{\sqrt{3}}{2}$

$\sin(60^\circ)=\frac{\sqrt{3}}{2}$

$\frac{120}{60}=2$

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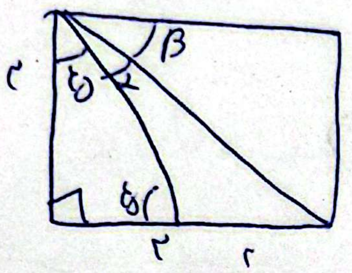

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$\alpha+\beta=45$

$\tan(\cancel{\alpha+\beta})=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$1-\frac{\tan\alpha}{2}=\tan\alpha+\frac{1}{2} \Rightarrow \tan\alpha=\frac{1}{3}$

$\Rightarrow \cot\alpha=3$

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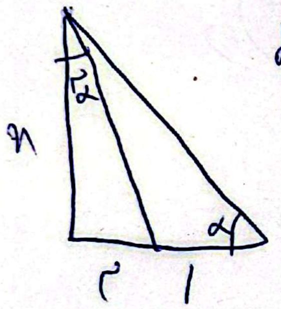

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$\tan2\alpha=\frac{3}{x}$

$\tan\alpha=\frac{x}{3}$

$\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$

$\frac{3}{x}=\frac{\frac{2x}{3}}{1-\frac{x^2}{9}}, \quad \frac{18-3x^2}{9}=\frac{2x^2}{3}$

$\rightarrow x^2=\frac{9}{4} \Rightarrow x=\frac{3}{2}$

$\Rightarrow \cot\alpha=\frac{3}{\frac{3}{2}}=2$

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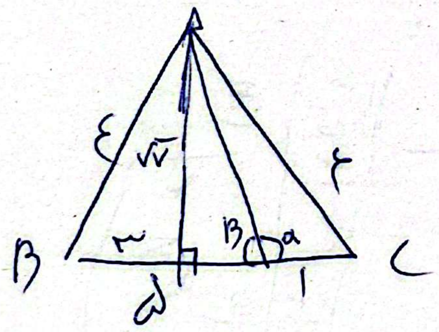

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$\alpha+\beta=\pi \qquad \tan(\pi-\alpha)=-\tan\alpha=\tan\beta$

$\tan\beta=\frac{\sqrt{7}}{2} \Rightarrow \tan\alpha=-\frac{\sqrt{7}}{2}$

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$1+\sin^2x=\frac{4}{3} \qquad \sin^2x=\frac{1}{3}$

$1-\sin^2x=\frac{2}{3}=\cos^2x \qquad \tan^2x=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{2}$

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$\frac{(1-\cos^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{(1-\sin^2\alpha)^2+4\sin^2\alpha}{1+\sin^2\alpha}$

$\frac{\cos^4\alpha+2\cos^2\alpha+1}{1+\cos^2\alpha}-\frac{\sin^4\alpha+2\sin^2\alpha+1}{1+\sin^2\alpha}=\cos^2\alpha-\sin^2\alpha=\cos2\alpha$

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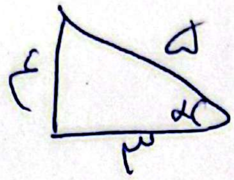
$\sin(\frac{\pi}{2}+\alpha)\cos(\frac{3\pi}{2}-\alpha)+\cot\alpha$

$\cos\alpha \qquad -\sin\alpha$

$=(\cos\alpha)(-\sin\alpha)+\cot\alpha$

$\frac{3}{5}\times-\frac{4}{5}+\frac{3}{4}=\frac{27}{100}$

$\cot\alpha=\frac{3}{4}$



$\Rightarrow \cos\alpha=\frac{3}{5}$

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$\sin^2\alpha=\frac{1-\cos 2\alpha}{2}$

$\sqrt{\frac{2-\sqrt{3}}{4}}=\sin\frac{\pi}{12} \qquad \sqrt{\frac{2+\sqrt{3}}{4}}=\cos\frac{\pi}{12}$

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$3\cos\frac{\pi}{3}+\sqrt{2}\left(\sin\frac{\pi}{12}-\cos\frac{\pi}{12}\right)$

$(\sin x-\cos x)^2=1-2\sin x\cos x=1-\frac{1}{2}=\sqrt{\frac{1}{2}}=|\sin x-\cos x|$

$\sin x-\cos x=-\frac{\sqrt{2}}{2}$

$\frac{3}{2}-1=\frac{1}{2}$

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$\tan\alpha=\frac{2\tan(\frac{\alpha}{2})}{1-(\tan\frac{\alpha}{2})^2}=\frac{8}{15}$

کل را بر cos تقسیم کنیم

$\Rightarrow \frac{\frac{\tan\alpha}{\cos\alpha}-\tan\alpha}{\tan\alpha-1}=\frac{\frac{\frac{8}{15}}{\frac{15}{17}}-\frac{8}{15}}{\frac{8}{15}-1}=\frac{-14}{105}$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}=1+\frac{64}{225}=\frac{289}{225}=\frac{1}{\cos\alpha}=\cos\alpha=\frac{15}{17}$

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$0<\frac{\cot\alpha}{\sin}\Rightarrow 0<\frac{\cos\alpha}{\sin^2\alpha}\Rightarrow\cos\alpha>0$

$0<\sin\alpha(\underbrace{\cos\alpha-1}_{\ominus})\Rightarrow\sin\alpha<0$ } ناحیه چهارم

$0<\cos\alpha<1\Rightarrow -1<\cos\alpha-1<0$