۱

مثلث ABC با اضلاع $۶, \sqrt{۳}$

زاویه α

$\overset{\triangle}{S} = ۴/۵$

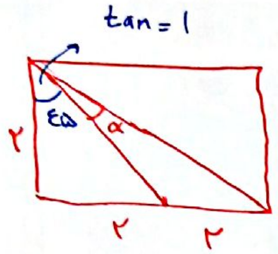
$$S = \frac{۱}{۲}\, ۶ \times \sqrt{۳} \times \sin\alpha = ۴/۵$$

$$\rightarrow ۶\sqrt{۳} \times \sin\alpha = ۹$$

$$\sin\alpha = \frac{\sqrt{۳}}{۲} \rightarrow \alpha \begin{cases} ۶۰^\circ \\ ۱۲۰^\circ \end{cases}$$

$$\frac{\alpha_{max}}{\alpha_{min}} = ? \quad \frac{۱۲۰}{۶۰} = ۲$$

۲



$$\tan(\alpha + ۴۵^\circ) = ۲ \rightarrow \tan(۲\alpha + ۹۰) = \frac{۲\tan(\alpha + ۴۵)}{۱ - \tan^۲(\alpha + ۴۵)} = \frac{۴}{۳}$$

$$\tan\left(\alpha + \frac{\pi}{۴}\right) = ۲ \qquad \tan\left(۲\alpha + \frac{\pi}{۲}\right) = \frac{۴}{۳}$$

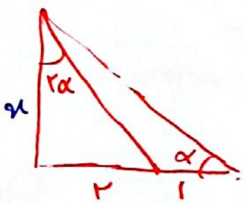
$$-\cot(۲\alpha) = \frac{۴}{۳} \qquad \tan ۲\alpha = \frac{۲\tan\alpha}{۱ - t^۲\alpha} = \frac{-۳}{۴}$$

$$\cot\alpha ? \quad \frac{۱}{\tan\alpha} = \frac{۱}{۳}$$

$$۳t^۲ - ۸t - ۳ = ۰$$

$$t \begin{cases} ۳ \\ \frac{-۱}{۳} \times \end{cases}$$

۳



$$\tan ۲\alpha = \frac{۲}{x} \qquad \tan ۲\alpha = \frac{۲\tan\alpha}{۱ - \tan^۲\alpha}$$

$$\tan\alpha = \frac{x}{۳}$$

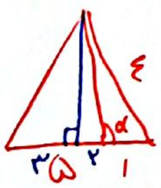
$$\frac{۲}{x} = \frac{\frac{۲x}{۳}}{۱ - \frac{x^۲}{۹}} \qquad \frac{۲}{x} = \frac{۶x}{۹ - x^۲}$$

$$\hookrightarrow ۸x^۲ = ۱۸$$

$$x^۲ = \frac{۱۸}{۸} \quad x = \frac{۳}{۲}$$

$$\cot\alpha = ? \quad \frac{۱}{\tan\alpha} = \frac{۱}{\frac{۲}{۳}} = ۲$$

۴



$$\rightarrow \tan(\pi - \alpha) = \frac{\sqrt{۷}}{۲}$$

$$\tan(-\alpha) = \frac{\sqrt{۷}}{۲}$$

$$\tan\alpha = -\frac{\sqrt{۷}}{۲}$$

تساوی الساقین است

$$\tan\alpha = ? \quad \frac{-\sqrt{۷}}{۲}$$

۵

$$۲\sin^۲ x + \cos^۲ x = \frac{۴}{۳}$$

$$۱ + \sin^۲ x = ۱ + \frac{۱}{۳}$$

$$\sin^۲ x = \frac{۱}{۳} \qquad \cos^۲ x = \frac{۲}{۳}$$

$$\sin x = \pm\frac{\sqrt{۳}}{۳} \qquad \cos x = \pm\frac{\sqrt{۶}}{۳}$$

$$\tan^۲ ? \quad \frac{\sin^۲}{\cos^۲} = \frac{\frac{۱}{۳}}{\frac{۲}{۳}} = \frac{۱}{۲}$$

۱۶

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2}=?\quad \frac{(1+c^2)^2}{1+c^2}-\frac{(1+s^2)^2}{1+s^2}=1+c^2-1-s^2$$

$$\sin^4-4\sin^2+4=(s^2-2)^2=(-1-c^2)^2=(1+c^2)^2$$

$\hookrightarrow = c^2-s^2=C2\alpha$

$\cos(2\alpha)$

۱۷

$\tan\alpha=\frac{4}{3} \rightarrow \cot\alpha=\frac{3}{4}$ $\quad \tan+\cot=\frac{1}{\sin\cos}=\frac{25}{12}$

ربع سوم

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right)-\tan\left(\alpha-\frac{3\pi}{2}\right)$$

$+\cos\alpha \times -\sin\alpha \quad -(-\cot\alpha)$

$$-\sin\cos+\cot\alpha=\frac{-12}{25}+\frac{3}{4}=\frac{27}{100}$$

۱۸

$x=\frac{\pi}{12}$

$$\sin\frac{\pi}{12}=\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=S\frac{\pi}{3}C\frac{\pi}{4}-S\frac{\pi}{4}C\frac{\pi}{3}=\frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=C\frac{\pi}{3}C\frac{\pi}{4}+S\frac{\pi}{3}S\frac{\pi}{4}=\frac{\sqrt{6}+\sqrt{2}}{4}$$

$3\cos 4x+\sqrt{2}\sin x-\sqrt{2}\cos x$

$$3\cos\frac{\pi}{3}+\sqrt{2}\sin\frac{\pi}{12}-\sqrt{2}\cos\frac{\pi}{12}=\frac{3}{2}+\sqrt{2}\left(\sin\frac{\pi}{12}-\cos\frac{\pi}{12}\right)=\frac{3}{2}+\sqrt{2}\left(\frac{-\sqrt{2}}{2}\right)$$

$$\hookrightarrow=\frac{1}{2}$$

۱۹

$\tan\frac{\alpha}{2}=\frac{1}{4}$ $\quad \frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}$ $\quad \frac{\frac{8}{15}-8k}{8k-15k}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{16}{255}}{\frac{-7}{17}}=\frac{-16}{105}$

$$\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}$$

$$\hookrightarrow=\frac{\frac{1}{2}}{1-\frac{1}{16}}=\frac{8}{15}$$

$\sin=8k$ $\quad 289k^2=1$

$\cos=15k$ $\quad k=\sqrt{\frac{1}{289}}=\frac{1}{17}$

۲۰

$2\sin\alpha<\sin2\alpha \rightarrow 2\sin<2\sin\cos$

$\begin{cases}\cos>1 & \times\\ \sin,\cos<0 & \checkmark\end{cases}$

ربع سوم

$\frac{\cot\alpha}{\sin\alpha}>0$

α = کدام ربع؟