امیرصدرا ۱۱ تجربی A تکلیف شماره ۲۷

۲۰

۱- (۲)

مساحت = ۴٫۵

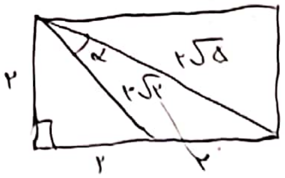
$S = \frac{1}{2}\sin\alpha \times 6 \times \sqrt{3} = 4.5$

$\sin\alpha = \frac{9 \times 2 \times \sqrt{3}}{6 \times \sqrt{3} \times 2} = \frac{\sqrt{3}}{2}$

$\sin\alpha \begin{cases} \alpha = 120^\circ \\ \alpha = 60^\circ \end{cases}$

$\frac{max}{min} = \frac{120}{60} = 2$ ✓

۲- (۲)



$\cot\alpha = ?$

$2 = \sqrt{2\sqrt{2}^2 + 2\sqrt{5}^2 - 2(2\sqrt{2} \times 2\sqrt{5})\cos\alpha}$

$4 = 28 - 8\sqrt{10}\cos\alpha$

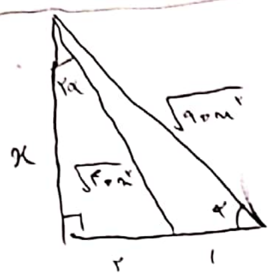
$8\sqrt{10}\cos\alpha = 24$

$\Rightarrow \cos\alpha = \frac{3}{\sqrt{10}}$

$\sin\alpha = \sqrt{1 - \frac{9}{10}} = \frac{1}{\sqrt{10}}$

$\cot\alpha = \frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}} = 3$ ✓

۳- (۲)



$\cot\alpha = \frac{3}{x} \quad \sin\alpha = \frac{x}{\sqrt{9+x^2}} \quad \cos\alpha = \frac{3}{\sqrt{9+x^2}}$

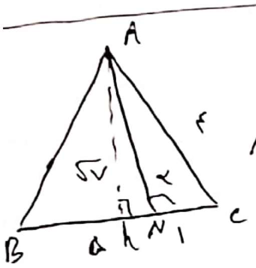
$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{2\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha} = \frac{2 \times \frac{x}{\sqrt{9+x^2}} \times \frac{3}{\sqrt{9+x^2}}}{\frac{9}{9+x^2} - \frac{x^2}{9+x^2}} = \frac{\frac{6x}{9+x^2}}{\frac{9-x^2}{9+x^2}} = \frac{6x}{9-x^2}$

$\tan 2\alpha = \frac{3}{x}$

$\frac{6x}{9-x^2} = \frac{3}{x} \Rightarrow 6x^2 = 18 - 2x^2,\ 8x^2 = 18 \rightarrow x^2 = \frac{9}{4},\ x = \pm\frac{3}{2}$

$\cot\alpha = \frac{3}{x} = 3 \times \frac{2}{3} = 2$ ✓

۴- (۲)



$hC = 3$

$Ah = \sqrt{16 - 9} = \sqrt{7}$

$AN = \sqrt{7 + 4} = \sqrt{11}$

$4 = \sqrt{1^2 + \sqrt{11}^2 - 2\sqrt{11}\cos\alpha}$

$16 = 12 - 2\sqrt{11}\cos\alpha$

$2\sqrt{11}\cos\alpha = -4$

$\cos\alpha = \frac{-2}{\sqrt{11}}$

$\sin\alpha = \sqrt{1 - \frac{4}{11}} = \frac{\sqrt{7}}{\sqrt{11}}$

$\tan\alpha = \frac{\frac{\sqrt{7}}{\sqrt{11}}}{\frac{-2}{\sqrt{11}}} = \boxed{\frac{-\sqrt{7}}{2}}$ ✓

۵- (۲)

$2\sin^2 x + \cos^2 x = \frac{4}{3}$

$\sin^2 x + \cos^2 x + \sin^2 x = \frac{4}{3}$ (1)

$\sin^2 x = \frac{1}{3}$

$\cos^2 x = 1 - \sin^2 = \frac{2}{3}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ ✓

۶- (۲)

$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} \longrightarrow 2 - \sin^2\alpha - (2 - \cos^2\alpha) = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$ ✓

$\frac{\sin^4\alpha + 4(1 - \sin^2\alpha)}{1 + (1 - \sin^2\alpha)} = \frac{(2 - \sin^2\alpha)^2}{2 - \sin^2\alpha} = 2 - \sin^2\alpha$

به این ترتیب قسمت

دوم هم $2 - \cos^2\alpha$ می‌شود

$\tan\alpha = \frac{4}{3}$

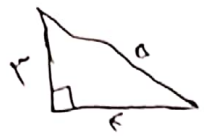
۷- (۲)

$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha-\frac{3\pi}{2}\right) = ?$

$\cos\alpha \quad , \quad -\sin\alpha \quad , \quad -(-\cot\alpha) = -\sin\alpha\cos\alpha + \cot\alpha$

$-\frac{3}{5}\times\frac{4}{5} + \frac{3}{4} = \frac{-12}{25} + \frac{3}{4}$

$-\frac{48}{100} + \frac{75}{100} = \boxed{0/27}$ ✓



$\frac{\pi}{12} = 15^\circ \qquad 2\cos 60^\circ + \sqrt{2}\sin 15^\circ - \sqrt{2}\cos 15^\circ$

۸- (۲)

$\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ\cos 30^\circ - \sin 30^\circ\cos 45^\circ = \frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} - \frac{1}{2}\times\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ\cos 30^\circ + \sin 45^\circ\sin 30^\circ = \frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\times\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

$2\times\frac{1}{2} + \sqrt{2}\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) - \sqrt{2}\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) = 1 + \frac{\sqrt{12}-\sqrt{4}-\sqrt{12}-\sqrt{4}}{4\sqrt{2}} = 1 - \frac{4}{4\sqrt{2}} = 1 - \frac{1}{\sqrt{2}}$ ✓

۹- (۲)

$\frac{\sin\alpha}{\cos\frac{\alpha}{2}} = \frac{1}{4} \qquad (4\sin^2\frac{\alpha}{2})^2 + \cos^2\frac{\alpha}{2} = 1$

$\sin\frac{\alpha}{2} = \frac{1}{\sqrt{17}} \quad , \quad \cos\frac{\alpha}{2} = \frac{4}{\sqrt{17}}$

$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = 2\times\frac{1}{\sqrt{17}}\times\frac{4}{\sqrt{17}} = \frac{8}{17}$

$\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} = \frac{16}{17} - \frac{1}{17} = \frac{15}{17}$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{15}{17} - \frac{8}{17}} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{-7}{17}}$

$\frac{8\times 17 - 8\times 15}{15\times 17}\times\frac{17}{-7} = \frac{16}{15}\times\frac{-1}{7}$

$= \boxed{\frac{-16}{105}}$ ✓

۱۰- (۲)

$2\sin\alpha < \sin 2\alpha$

$2\sin\alpha < 2\sin\alpha\cos\alpha$

$\cos\alpha > 0$

اگر $\cos\alpha = 1$ باشد $\sin\alpha$ عددی صفر بوده که با شرط این اتفاق نشده ← $\sin\alpha < 0$

با عددگذاری 45° نادرست و جواب نمی دهد

اما با -45° جواب می دهد

$\cos\alpha > 0$, $\sin\alpha < 0$ ← ربع چهارم ✓