آرسین ایزدی   کلاس A   تکلیف ۲۸

۱-

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \quad , \quad \cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$$

ناحیه اول

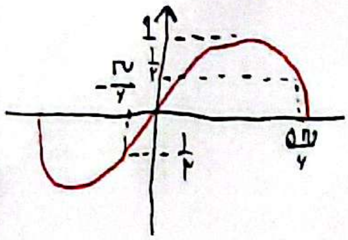
$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \to \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \to \sin\alpha > 0$$

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \to \frac{1}{|\cos\alpha|} - \frac{1-\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \to \frac{\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \to \cos\alpha > 0$$

۲-

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \quad , \quad \sin 2x = \frac{m-1}{4}$$

$$\to -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \xrightarrow{t=2x} -\frac{\pi}{6} < t < \frac{5\pi}{6}$$



$$\to -\frac{1}{2} < \sin t \leq 1 \to -\frac{1}{2} < \frac{m-1}{4} \leq 1 \to m \in (-1, 5]$$

۳-

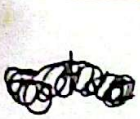
$$\tan x + \cot x = -3 \qquad 4\pi > 4x > 3\pi$$

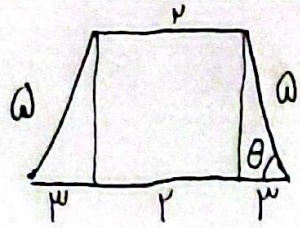
$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = -3 \to \sin x \cos x = -\frac{1}{3} \qquad (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\sin x + \cos x = -\frac{1}{\sqrt{3}}$$

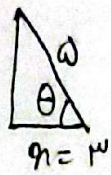
$$\frac{1}{\sin^3 x + \cos^3 x} = \frac{-1}{(\sin x + \cos x)(\sin^2 x - \sin x\cos x + \cos^2 x)} = \frac{1}{-\frac{1}{\sqrt{3}}(1+\frac{1}{3})} = \frac{3}{4}\sqrt{3}$$

-۴





$\overset{r=y}{\Longrightarrow}$



$\rightarrow \cos\theta = \frac{۴}{۵} = \frac{x}{۵} \rightarrow x = ۳$

$$S = \frac{(۲+۸)\times ۴}{۲} = \boxed{۲۰}$$

-۵

$$\tan(۲۸\omega)\tan(-۱۲\omega) - \sin(۱۰۹\omega)\cos(۲۰۵\omega) = -\cot(۱\omega)\times\tan(۱\omega) + \sin۱\omega \times -\sin۱\omega$$

$$= -۱ + \sin^{۲}۱\omega = -\cos^{۲}۱\omega \Rightarrow \boxed{K=-1}$$

-۶

$$A = \sqrt{۳}\cos(۲۱۰)\sin(۲۴۳) - \sqrt{۲}\sin(۱۳۵)\cos(۱۵۳)$$

$$= \sqrt{۳}\cos(۱۸۰+۳۰)\sin(۲۷۰-۲۷) - \sqrt{۲}(۹۰+۴۵)\cos(۱۸۰-۲۷) = \sqrt{۳}(-\cos۳۰)(-\cos۲۷) - \sqrt{۲}(\sin۴۵)\times$$

$$= \sqrt{۳}\left(\frac{\sqrt{۳}}{۲}\right)\cos^{۲}۷ + \sqrt{۲}\left(\frac{\sqrt{۲}}{۲}\right)\cos^{۲}۷ = \frac{۵}{۲}\cos^{۲}۷ \Rightarrow \boxed{\text{برابر } \frac{۵}{۲}}$$

-۷

$$f(x) = ۱۶\cos^{۲}(۲x)\cos^{۲}(۴x)\cos^{۲}(۱۶x)\cos^{۲}(۳۲x)$$

$$f\left(\frac{\pi}{۳۶}\right) = ۱۶\cos^{۲}\left(\frac{\pi}{۱۸}\right)\cos^{۲}\left(\frac{\pi}{۹}\right)\cos^{۲}\left(\frac{\pi}{۴}\right)\cos^{۲}\left(\frac{۸\pi}{۳}\right) = ۱۶\cos^{۲}\frac{\pi}{۱۸}\left(\frac{\sqrt{۳}}{۲}\right)^{۲}\left(\frac{۱}{۲}\right)^{۲}\left(-\frac{۱}{۲}\right)^{۲} = \frac{۳}{۴}\cos^{۲}\frac{\pi}{۱۸}$$

$\cos^{۲}\alpha = \frac{۱}{۲}(۱+\cos۲\alpha)$

$$\rightarrow f\left(\frac{\pi}{۳۶}\right) = \frac{۳}{۴}\times\frac{۱}{۲}\left(۱+\cos\frac{۲\pi}{۱۸}\right) = \frac{۳}{۸}\left(۱+\frac{\sqrt{۳}}{۲}\right) = \boxed{\frac{۴+۳\sqrt{۳}}{۱۶}}$$

-۸

$$\frac{۱-\sin x}{۱+\sin x} = ۲ \rightarrow ۱-\sin x = ۲(۱+\sin x) \rightarrow ۳\sin x = -۱ \rightarrow \sin x = -\frac{۱}{۳}$$

$$-\sin x = \frac{۲\tan\frac{x}{۲}}{۱+\tan^{۲}\frac{x}{۲}} \rightarrow -\frac{۱}{۳} = \frac{۲\tan\frac{x}{۲}}{۱+\tan^{۲}\frac{x}{۲}} \rightarrow ۳\tan^{۲}\frac{x}{۲} + ۱۰\tan\frac{x}{۲} + ۳ = ۰$$

$$\tan\frac{x}{۲} = \frac{-۱۰\pm\sqrt{۶۴}}{۶} = \frac{-۱۰\pm۸}{۶} \rightarrow \tan\frac{x}{۲} \begin{cases} = -۳ \checkmark \\ = -\frac{۱}{۳} \times \end{cases} \rightarrow \boxed{\tan\frac{x}{۲} = -۳}$$

-۹

$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{\sin^2\theta+1-\cos^2\theta}{(1-\cos\theta)(\sin\theta)}=\frac{2\sin^2\theta}{(1-\cos\theta)\sin\theta}=\frac{2\sin\theta}{1-\cos\theta}=\frac{4\times\sin\frac{\theta}{2}\times\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$\rightarrow \frac{2\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}=2\cot\frac{\theta}{2}\Rightarrow k=2$$

-۱۰

$$A=\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\left(3\pi-\frac{\pi}{4}+\alpha\right)=-\cos\left(\alpha-\frac{\pi}{4}\right)=-\left(\cos\alpha\cos\frac{\pi}{4}+\sin\alpha\sin\frac{\pi}{4}\right)$$

$$\rightarrow -\frac{\sqrt{2}}{2}(\sin\alpha+\cos\alpha)\quad,\quad \cos^2\alpha=1-\sin^2\alpha=1-\frac{2}{100}=\frac{49}{50}\rightarrow\cos\alpha=\frac{-7}{5\sqrt{2}}=-\frac{7\sqrt{2}}{10}$$

$$\hookrightarrow A=-\frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{10}-\frac{7\sqrt{2}}{10}\right)=\frac{4}{5}$$