ایلیا مسلمی — یازدهم سپهر A — تکلیف شماره ۲۸

Subject:

Year: Month: Day: ( ) page: ( )

۱-

$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow |\sin\alpha| = \sin\alpha \Rightarrow \sin\alpha \geq 0$ (I)

$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow |\cos\alpha| = \cos\alpha$

$\Rightarrow \cos\alpha \geq 0$ (II)

(I) $\cap$ (II) $\rightarrow$ ناحیه اول

۲-

$-\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin 2x \leq 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1$

$\Rightarrow -2 < m-1 \leq 4 \Rightarrow \boxed{-1 < m \leq 5}$

۳-

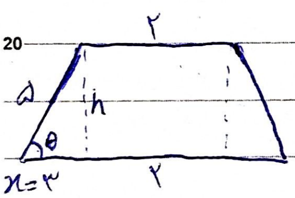
$\frac{3\pi}{4} < x < \pi \Rightarrow \sin x + \cos x < 0 \qquad \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = -3$

$\Rightarrow \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = -3 \Rightarrow \frac{1}{\sin x \cos x} = -3 \Rightarrow \sin x \cos x = -\frac{1}{3}$

$\frac{1}{\sin^3 + \cos^3} = \frac{1}{(\sin + \cos)(\sin^2 + \cos^2 - \sin\cos)} = \frac{1}{\frac{4}{3}(\sin + \cos)}$

$= \frac{1}{\frac{4}{3}\sqrt{\sin^2 + \cos^2 + 2\sin\cos}} = \frac{1}{\frac{4}{3} \times (-\frac{1}{\sqrt{3}})} = \boxed{\frac{3\sqrt{3}}{4}}$

۴-



$\cos\theta = 0.6 \Rightarrow x = 5 \times 0.6 = 3 \Rightarrow h = 4$

$S = \frac{4(2+8)}{2} = \boxed{20}$

۵-

$\tan(285) = \tan(\frac{3\pi}{2} + 15) = -\cot 15$

$\tan(-165) = \tan(-\pi + 15) = \tan 15$

$\sin(1095) = \sin(6\pi + 15) = \sin 15$

$\cos(\frac{3\pi}{2} - 15) = -\sin 15$

$\Rightarrow \tan(285)\tan(-165) - \sin(1095)\cos(255) = K\cos^2 15$

$(-\cot 15)(\tan 15) - (\sin 15)(-\sin 15) = -1 - (-\sin^2 15) = -\cos^2 15$

$-\cos^2 15 = K\cos^2 15 \Rightarrow \boxed{K = -1}$

$\sin(۲۴۳) = \sin(\pi + ۶۳) = -\sin ۶۳ = -\cos ۲۷$ —۶

$\cos ۱۵۳ = \cos(\pi - ۲۷) = -\cos ۲۷$

$A = \sqrt{۳} \times \left(-\frac{\sqrt{۳}}{۲}\right) \times (-\cos ۲۷) - \sqrt{۲} \times \frac{\sqrt{۲}}{۲} \times (-\cos ۲۷) = \cos ۲۷\left(\frac{۳}{۲}+۱\right) = \frac{۵}{۲}\cos ۲۷$

$\Rightarrow \frac{A}{\cos ۲۷} = \frac{\frac{۵}{۲}\cos ۲۷}{\cos ۲۷} = \boxed{\frac{۵}{۲}}$

$f\left(\frac{\pi}{۳۶}\right) = ۱۶\, \underbrace{\cos^۲\left(\frac{\pi}{۱۲}\right)}_{\frac{۱+\cos\frac{\pi}{۶}}{۲}} \underbrace{\cos^۲\left(\frac{\pi}{۶}\right)}_{\frac{۳}{۴}} \underbrace{\cos^۲\left(\frac{\pi}{۳}\right)}_{\frac{۱}{۴}} \underbrace{\cos^۲\left(\frac{۲\pi}{۳}\right)}_{\frac{۱}{۴}}$ —۷

$= ۱۶\left(\frac{۲+\sqrt{۳}}{۴}\right) \times \frac{۳}{۴} \times \frac{۱}{۴} \times \frac{۱}{۴} = \frac{۳(۲+\sqrt{۳})}{۱۶} = \frac{۶+۳\sqrt{۳}}{۱۶}$

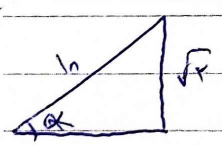
$۱ - \sin x = ۴ + ۴\sin x \Rightarrow \sin x = -\frac{۳}{۵}$ —۸

$\sin x = \frac{۲\tan\frac{x}{۲}}{۱+\tan^۲\frac{x}{۲}} = -\frac{۳}{۵} \Rightarrow ۱۰\tan\frac{x}{۲} = -۳ - \tan^۲\frac{x}{۲}$

$\Rightarrow \tan^۲\frac{x}{۲} + ۱۰\tan\frac{x}{۲} + ۳ = ۰ \Rightarrow \tan\frac{x}{۲} \begin{cases} -\frac{۱}{۳} \\ \boxed{-۳} \rightarrow \text{مقدار صحیح} \end{cases}$

$\frac{\sin^۲ + ۱ - \cos^۲}{(۱-\cos)(\sin)} = \frac{\sin^۲ + \sin^۲ + \cos^۲ - \cos^۲}{(۱-\cos)(\sin)} = \frac{۲\sin^{\cancel{۲}}}{(۱-\cos)\cancel{(\sin)}} = \frac{۲\sin}{۱-\cos}$ —۹

$\Rightarrow \frac{۲\sin\theta}{۱-\cos\theta} = k\,\frac{\cos\frac{\theta}{۲}}{\sin\frac{\theta}{۲}}$ $\quad \frac{\sin\theta}{۱-\cos\theta} = \cot\frac{\theta}{۲} \rightarrow \boxed{k = ۲}$



$\cos\alpha = \frac{\sqrt{۹۸}}{۱۰}$

$\cos\left(\frac{۱۱\pi}{۴} + \alpha\right) = \cos\left(\frac{۱۱\pi}{۴}\right)\cos\alpha - \sin\left(\frac{۱۱\pi}{۴}\right)\sin\alpha$ —۱۰

$= \left(\left(\frac{-\sqrt{۲}}{۲}\right) \times \left(-\frac{\sqrt{۹۸}}{۱۰}\right)\right) - \left(\frac{\sqrt{۲}}{۱۰} \times \frac{\sqrt{۲}}{۲}\right) =$

$\frac{\sqrt{۱۹۶}}{۲۰} - \frac{۲}{۲۰} = \frac{۱۲}{۲۰} = \boxed{\frac{۳}{۵}}$