رادین ملکیان یازدهم A

-۱

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|}$$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow |\cos\alpha| = \cos\alpha \Rightarrow \cos\alpha > 0$$

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha > 0$$

α ناحیه اول است

-۲

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \underbrace{\sin 2x}_{\frac{m-1}{4}} < 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} < 1$$

$$\Rightarrow -1 < m < 5$$

-۳

$$\tan x + \cot x = \frac{1}{\sin x\cos x} = -3 \Rightarrow \sin x \cdot \cos x = -\frac{1}{3}$$

$$3\pi < 4x < 4\pi \Rightarrow \frac{3}{4}\pi < x < \pi \qquad \sin x + \cos x < 0$$

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{3} \Rightarrow (\sin x + \cos x)^2 = \frac{1}{3}$$

$$\Rightarrow \sin x + \cos x = -\sqrt{\frac{1}{3}} \Rightarrow \sin^3 x + \cos^3 x = (\sin x + \cos x)^3 - 3\sin x\cos x(\sin x + \cos x)$$

$$= -\frac{1}{3}\sqrt{\frac{1}{3}} - 3\left(-\frac{1}{3}\right)\left(-\frac{1}{\sqrt{3}}\right) = -\frac{1}{3}\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{3}} = -\frac{4}{3}\sqrt{\frac{1}{3}}$$

$$\Rightarrow \frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{-\frac{4}{3}\sqrt{\frac{1}{3}}} = \frac{1}{-\sqrt{\frac{16}{27}}} = -\frac{\sqrt{27}}{\sqrt{16}} = -\frac{3}{4}\sqrt{3}$$

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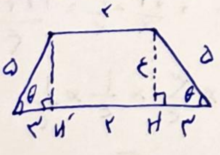

$$\overset{\triangle}{ABH}: \quad \cos\theta = \frac{BH}{AB} = \frac{BH}{5} = \frac{6}{10} \Rightarrow BH = 3$$

فیثاغورس $\Rightarrow AH^2 + BH^2 = AB^2 \Rightarrow AH^2 + 9 = 25 \Rightarrow AH^2 = 16 \Rightarrow AH = 4$

$$\left.\begin{array}{l} AB = DC \\ \hat{H}_1 = \hat{H}'_1 = 90 \\ C = B = \theta \end{array}\right\} \xRightarrow{\text{وز}} \overset{\triangle}{ABH} \cong DCH' \Longrightarrow CH' = BH = 3$$

$$S = \frac{AD + \overset{\uparrow\, CH' + H'H + BH}{BC}}{2} \times AH = \frac{2+8}{2} \times 4 = 20$$

$$\left.\begin{array}{l} AD \parallel HH' \\ \left.\begin{array}{l} AH \perp BC \\ DH' \perp BC \end{array}\right\} AH \parallel DH' \\ \hat{H}_2 = 90 \end{array}\right\} \underset{\text{مستطیل}}{ADH'H} \Rightarrow AD = HH' = 2$$

$$\tan(285^\circ)\tan(-165) - \sin(1095)\cos(255) = -\underbrace{\cot(15)\tan(15)}_{-1} - \sin(15)(-)(\sin 15)$$

$$\tan(285) = \tan(270+15) = \tan\left(\frac{3\pi}{2}+15\right) = -\cot 15$$

$$\tan(-165) = \tan(15-180) = \tan(15-\pi) = \tan 15$$

$$\sin(1095) = \sin(1080+15) = \sin(6\pi+15) = \sin 15$$

$$\cos(255) = \cos(270-15) = \cos\left(\frac{3\pi}{2}-15\right) = -\sin 15$$

$$\Rightarrow$$

$$-1+\sin^2(15) = -(1-\sin^2 15) = -\cos^2 15 = k\cos^2(15) \Rightarrow k=-1$$

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$$A = \sqrt{3}\cos(210)\underbrace{\sin(243)}_{-\cos(27)} - \sqrt{2}\sin(135)\underbrace{\cos(153)}_{-\cos(27)}$$

$$= \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)(-)\cos(27) - \sqrt{2}\left(\frac{\sqrt{2}}{2}\right)(-)\cos(27) = \frac{3}{2}\cos(27) + \cos(27) = \frac{5}{2}\cos(27)$$

$$\frac{\frac{5}{2}\cos(27)}{\cos(27)} = \frac{5}{2}$$

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$$f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{\pi}{12}\right)\times\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{2\pi}{3}\right)$$

$$\cos^2\left(\frac{2\pi}{3}\right) = -\frac{1}{2}\times-\frac{1}{2} = \frac{1}{4}, \quad \cos^2\left(\frac{\pi}{3}\right) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

$$\cos^2\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{3}{4} \qquad \cos^2\left(\frac{\pi}{12}\right) = \cos^2(15) = \cos^2(45-30)$$

$$\cos(45-30) = \cos(45)\cos(30) + \sin(45)\sin(30) \Rightarrow \cos(45-30) = \frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\times\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\Rightarrow \cos^2(15) = \frac{6+2+2\sqrt{12}}{4\times4} = \frac{2+\sqrt{3}}{4} \qquad \Rightarrow f(a) = 16\times\frac{2+\sqrt{3}}{4}\times\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}$$

$$= \frac{3\times(2+\sqrt{3})}{4\times4} = \frac{6+3\sqrt{3}}{16}$$

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$$1-\sin x = 4+4\sin x \Rightarrow 5\sin x = -3 \Rightarrow \sin x = -\frac{3}{5} \Rightarrow \sin^2 x = \frac{9}{25} \Rightarrow 1-\sin^2 = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm\frac{4}{5}$$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{-\frac{3}{5}}{1-\frac{4}{5}} = -3$$

$$\frac{\sin\theta}{1-\cos\theta} = \frac{۲\cos\frac{\theta}{۲}\sin\frac{\theta}{۲}}{۲\sin^{۲}\frac{\theta}{۲}} = \frac{\cos\frac{\theta}{۲}}{\sin\frac{\theta}{۲}} = \cot\frac{\theta}{۲}$$

$$\frac{\sin\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta} \Rightarrow \frac{\sin\theta(1+\cos\theta)}{1-\cos^{۲}\theta} = \frac{\sin\theta(1+\cos\theta)}{\sin^{۲}\theta} = \frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{۲}$$

$$\Rightarrow \frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = ۲\cot\left(\frac{\theta}{۲}\right) = k\cot\frac{\theta}{۲} \Rightarrow k=۲$$

$$\sin\alpha = \frac{\sqrt{۲}}{۱۰} \Rightarrow \sin^{۲}\alpha = \frac{۲}{۱۰} \Rightarrow 1-\sin^{۲}\alpha = \frac{۹۸}{۱۰۰} \Rightarrow \cos^{۲}\alpha = \frac{۹۸}{۱۰۰} \Rightarrow \cos\alpha = \pm\frac{۷\sqrt{۲}}{۱۰}$$ ۱۱۰

$$\xrightarrow{\alpha \text{ ربع دوم}} \cos\alpha = \frac{-۷\sqrt{۲}}{۱۰}$$

$$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right) = \cos\left(\frac{۱۱\pi}{۴}\right)\times\cos(\alpha) - \sin\left(\frac{۱۱\pi}{۴}\right)\sin(\alpha) = \left(-\frac{\sqrt{۲}}{۲}\times\frac{-۷\sqrt{۲}}{۱۰}\right) - \left(\frac{\sqrt{۲}}{۲}\times\frac{\sqrt{۲}}{۱۰}\right)$$

$$= \frac{۱۴}{۲۰} - \frac{۲}{۱۰} = \frac{۱۲}{۲۰} = \frac{۶}{۱۰}$$