۱- حالت اول

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \quad , \quad \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$\Rightarrow \cos\alpha = |\cos\alpha| \Rightarrow \cos\alpha > 0$; $\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|}$

$\Rightarrow \sin\alpha = |\sin\alpha| \Rightarrow \sin\alpha > 0$

۲-

$$-\frac{\pi}{12} < x < \frac{5\pi}{12}$$

$$-\frac{\pi}{6} < 2x < \frac{5\pi}{6}$$

$$-\frac{1}{2} < \sin 2x \leq 1 \qquad -\frac{1}{2} < \frac{1-m}{2} \leq 1 \qquad -2 < 1-m \leq 2$$

$$-1 < m \leq 3$$

$m \in (-1, 3]$

۳-

$\sin\alpha > 0 \quad \cos\alpha < 0$

$\frac{\pi}{2} < \alpha < \pi$; $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$

$\tan\alpha + \cot\alpha = -2 \Rightarrow \frac{1}{\sin\alpha\cos\alpha} = -2 \Rightarrow \sin\alpha\cos\alpha = -\frac{1}{2}$

$$\frac{1}{\sin^2\alpha + \cos^2\alpha} = \frac{1}{(\sin\alpha+\cos\alpha)(\sin^2\alpha+\cos^2\alpha - \sin\alpha\cos\alpha)} = \frac{1}{(\sin\alpha+\cos\alpha)\left(1+\frac{1}{2}\right)}$$

$(\sin\alpha+\cos\alpha)^2 = 1 + 2\sin\alpha\cos\alpha = \frac{1}{2}$

$\Rightarrow \sin\alpha+\cos\alpha = -\sqrt{\frac{1}{2}}$

زیرا در این ناحیه $\cos > \sin$

$$\Rightarrow \frac{1}{\frac{3}{2}\left(-\sqrt{\frac{1}{2}}\right)} = \frac{-2\sqrt{2}}{3}$$

۴-

$\cos\theta = \frac{x}{5} \Rightarrow x = 5\cos\theta$

$S_{\text{ذوزنقه}} = \frac{4 \times (7+2)}{2} = 20$

۵-

$$k\cos^2 15 = \tan(285)\tan(-195) - \sin(105)\cos(255)$$

$$-\cot(105)\tan(15) - \sin(15)\sin(15) = -1 + \sin^2 15$$

$$= -\cos^2 15 \Rightarrow k = -1$$

۶-

$$\sqrt{2}\cos(210)\sin(255) - \sqrt{2}\sin(135)\cos(150) =$$

$$\sqrt{2} \times -\frac{\sqrt{3}}{2} \times -\cos(75) - \sqrt{2} \times \frac{\sqrt{2}}{2} \times -\cos(75) = \frac{5}{2}\cos(75)$$

$$\frac{\frac{5}{2}\cos(75)}{\cos(75)} = \frac{5}{2} = 2.5$$

٨-

$$f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{2\pi}{24}\right)\cos^2\left(\frac{4\pi}{24}\right)\cos^2\left(\frac{8\pi}{24}\right)\cos^2\left(\frac{16\pi}{24}\right)$$

$$16\times\cos^2\left(\frac{\pi}{12}\right)\times\frac{1}{4}\times\frac{1}{4} = \frac{2+\sqrt{3}}{4} = \frac{8+4\sqrt{3}}{16}$$

$$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{1}{2}\times\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4} \Rightarrow \frac{8+4\sqrt{3}}{16} = \frac{2+\sqrt{3}}{4}$$

٧-

$$\frac{1-\sin\alpha}{1+\sin\alpha} = 2 \qquad 4+4\sin\alpha = 1-\sin\alpha$$

$$5\sin\alpha = -3$$

$$\sin\alpha = -\frac{3}{5} \Rightarrow \cos\alpha = -\frac{4}{5}$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\frac{9}{5}}{-\frac{3}{5}} = -3$$

٩-

$$k\cot\frac{\theta}{2} = \frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\cot\frac{\theta}{2} \Rightarrow k = 2$$

$$\cot\frac{\theta}{2} \qquad \frac{1}{\tan\frac{\theta}{2}} = \cot\frac{\theta}{2}$$

١٠-

$$\sin\alpha = \frac{\sqrt{3}}{10}$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\frac{3}{100} + \frac{91}{100} = 1 \Rightarrow \cos\alpha = -\frac{\sqrt{91}}{10}$$

$$\cos\left(\frac{11\pi}{4}+\alpha\right)$$

$$\frac{3\pi}{4}$$

$$= \cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha = \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = 0.6$$

$$-\frac{\sqrt{2}}{2}\times\frac{-\sqrt{91}}{10} - \frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{10}$$

$$+\frac{\sqrt{182}}{20} - \frac{\sqrt{6}}{20} = -\frac{1}{10}$$

$$+\frac{14}{20} = +\frac{7}{10}$$