پارسا جهانگیری / یازدهم تجربی A

۱۹,۷۵

(۲)

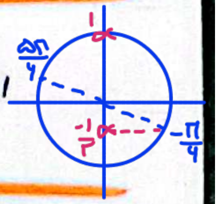
1- $\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|}$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow |\cos\alpha| = \cos\alpha \Rightarrow \boxed{\cos\alpha > 0}\ (I)$

$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow |\sin\alpha| = \sin\alpha \Rightarrow \boxed{\sin\alpha > 0}\ (II)$

$(II), (I) \Rightarrow$ α در ناحیه اول قرار دارد

۱,۷۵



2- $-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin 2x \leq 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1$; $\sin 2x = \frac{m-1}{4}$

$\Rightarrow -2 < m-1 \leq 4 \Rightarrow \boxed{-1 < m \leq 5}$ دقت!

3- $\tan x + \cot x = \frac{1}{\sin x \cos x} = -3 \Rightarrow \boxed{\sin x \cos x = -\frac{1}{3}}\ I$; $3\pi < 4x < 4\pi \Rightarrow \frac{3\pi}{4} < x < \pi \Rightarrow \sin x + \cos x < 0\ (III)$

(۲)

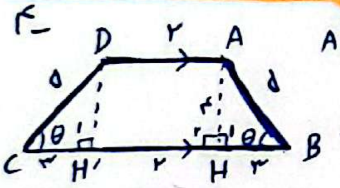
$\sin^2 x + \cos^2 x = 1 \overset{(I)}{\Rightarrow} \sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{3} \Rightarrow (\sin x + \cos x)^2 = \frac{1}{3} \overset{(III)}{\Rightarrow} \sin x + \cos x = -\sqrt{\frac{1}{3}}$

$\Rightarrow \sin^3 x + \cos^3 x = (\sin x + \cos x)^3 - 3\sin x\cos x(\sin x + \cos x)$

$= -\frac{1}{3}\sqrt{\frac{1}{3}} - 3(-\frac{1}{3})(-\sqrt{\frac{1}{3}}) = -\frac{1}{3}\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{3}} = -\frac{4}{3}\sqrt{\frac{1}{3}}$

$\Rightarrow \frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{-\frac{4}{3}\sqrt{\frac{1}{3}}} = \frac{1}{-\sqrt{\frac{16}{27}}} = -\frac{\sqrt{27}}{\sqrt{16}} = \boxed{-\frac{3}{4}\sqrt{3}}$

4-



(۲)

$A\hat{H}B$:

$\cos\theta = \frac{BH}{AB} = \frac{BH}{5} = \frac{6}{10} \Rightarrow BH = 3$

فیثاغورث $\Rightarrow AH^2 + BH^2 = AB^2 \Rightarrow AH^2 + 9 = 25 \Rightarrow AH^2 = 16 \Rightarrow AH = 4$

$AB = DC$, $\hat{H}_1 = \hat{H}'_1 = 90°$, $\hat{C} = \hat{B} = \theta$ } وز $\Rightarrow A\hat{B}H \cong D\hat{C}H'$ $\xrightarrow{\text{تساوی اضلاع متناظر}}$ $CH' = BH = 3$

$S = \frac{AD + BC}{2} \times AH = \frac{2+8}{2} \times 4 = \boxed{20}$; $BC = CH' + H'H + BH$

$AH \perp BC$, $DH' \perp BC$ } $\Rightarrow AH \parallel DH'$, $\hat{H}_2 = 90°$, $AD \parallel HH'$ } $\Rightarrow ADH'H$ مستطیل است $\Rightarrow AD = HH' = 2$

(۲)

5- $\tan(285°)\tan(-165°) - \sin(1095°)\cos(255°) \overset{(I)}{=} \underbrace{-\cot(15°)\tan(15°)}_{-1} - \sin(15°)(-)\sin(15°)$

$= -1 + \sin^2(15°) = -(1-\sin^2 15°) = -\cos^2 15°$

$\tan(285°) = \tan(270° + 15°) = \tan(\frac{3\pi}{2} + 15°) = -\cot 15°$

$\tan(-165°) = \tan(15° - 180°) = \tan(15° - \pi) = \tan 15°$

$\sin(1095°) = \sin(1080° + 15°) = \sin(6\pi + 15°) = \sin 15°$

$\cos(255°) = \cos(270° - 15°) = \cos(\frac{3\pi}{2} - 15°) = -\sin 15°$ } (I)

$\Rightarrow -\cos^2 15° = K\cos^2 15°$

$\Rightarrow \boxed{K = -1}$

پارسا جهانگیری / یازدهم پسر A

6- $A=\sqrt{3}\cos(210°)\underbrace{\sin(243°)}_{-\cos(27°)}-\sqrt{2}\sin(135°)\underbrace{\cos(153°)}_{-\cos(27°)}$ ۲

$=\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)(-)\cos(27°)-\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)(-)\cos(27°)$

$=\frac{3}{2}\cos(27°)+\cos(27°)=\frac{5}{2}\cos(27°)$ $\qquad \frac{\frac{5}{2}\cos(27°)}{\cos(27°)}=\boxed{\frac{5}{2}}$

7- $f(x)=16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x)$

$\xrightarrow{\times\frac{\sin^2(3x)}{\sin^2(3x)}} f(x)=\frac{16\cdot 4\left(\sin(3x)\cos(3x)\right)^2\cos^2(6x)\cos^2(12x)\cos^2(24x)}{4\sin^2(3x)}=\frac{4\sin^2(6x)\cos^2(6x)\cos^2(12x)\cos^2(24x)}{\sin^2(3x)}$ ۲

$\xlongequal{\times\frac{16}{16}}\frac{16\left(4\sin^2(12x)\cos^2(12x)\right)\cos^2(24x)}{16\sin^2(3x)}=\frac{4\sin^2(24x)\cos^2(24x)}{16\sin^2(3x)}=\frac{\sin^2(48x)}{16\sin^2(3x)}$

$\Rightarrow f\left(\frac{\pi}{36}\right)=\frac{\sin^2\left(\frac{4\pi}{3}\right)}{16\sin^2\left(\frac{\pi}{12}\right)}=\frac{\frac{3}{4}}{16\times\left(\frac{2-\sqrt{3}}{4}\right)}=\frac{3}{16(2-\sqrt{3})}\xlongequal{\times\frac{(2+\sqrt{3})}{(2+\sqrt{3})}}\frac{3(2+\sqrt{3})}{16(4-3)}=\boxed{\frac{6+3\sqrt{3}}{16}}$

$\sin^2\frac{\pi}{12}=\frac{1-\cos\frac{\pi}{6}}{2}=\frac{1-\frac{\sqrt{3}}{2}}{2}=\frac{2-\sqrt{3}}{4}$

8- $1-\sin x=4+4\sin x\Rightarrow 5\sin x=-3\Rightarrow\boxed{\sin x=-\frac{3}{5}}\Rightarrow\sin^2x=\frac{9}{25}\Rightarrow 1-\sin^2x=\frac{16}{25}$

$\Rightarrow\cos^2x=\frac{16}{25}\Rightarrow\cos x=\pm\frac{4}{5}$ ۲

$\tan\frac{x}{2}=\frac{\sin x}{1+\cos x}=\frac{-\frac{3}{5}}{1-\frac{4}{5}}=\frac{-\frac{3}{5}}{\frac{1}{5}}=\boxed{-3}$ $\qquad \xrightarrow{\text{در ربع سوم است}}\boxed{\cos x=-\frac{4}{5}}$

اثبات: $\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\tan\frac{x}{2}$

9- $\frac{\sin\theta}{1-\cos\theta}=\frac{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}=\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}=\cot\frac{\theta}{2}\quad(I)$ ۲

$\frac{\sin\theta}{1-\cos\theta}\xlongequal{\times\frac{1+\cos\theta}{1+\cos\theta}}\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}=\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}=\frac{1+\cos\theta}{\sin\theta}\overset{(I)}{=}\cot\frac{\theta}{2}$

$\Rightarrow\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\cot\frac{\theta}{2}=k\cot\frac{\theta}{2}\Rightarrow\boxed{k=2}$

10- $\sin\alpha=\frac{\sqrt{2}}{10}\Rightarrow\sin^2\alpha=\frac{2}{100}\Rightarrow 1-\sin^2\alpha=\frac{98}{100}\Rightarrow\cos^2\alpha=\frac{98}{100}\Rightarrow\cos\alpha=\pm\frac{7\sqrt{2}}{10}$

۲

$\xrightarrow{\alpha\text{ در ربع دوم است}}\boxed{\cos\alpha=-\frac{7\sqrt{2}}{10}}$

$\cos\left(\frac{11\pi}{4}+\alpha\right)=\underbrace{\cos\left(\frac{11\pi}{4}\right)}_{\cos\left(\frac{3\pi}{4}\right)}\cos(\alpha)-\underbrace{\sin\left(\frac{11\pi}{4}\right)}_{\sin\left(\frac{3\pi}{4}\right)}\sin(\alpha)=\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{7\sqrt{2}}{10}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{10}\right)$

$=\frac{14}{20}-\frac{2}{20}=\frac{12}{20}=\boxed{\frac{6}{10}}$