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1- $\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|}$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow |\cos\alpha| = \cos\alpha \Rightarrow \boxed{\cos\alpha > 0}\ (I)$

$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow |\sin\alpha| = \sin\alpha \Rightarrow \boxed{\sin\alpha > 0}\ (II)$

(I) و (II) $\Rightarrow$ α در ناحیه اول قرار دارد

2- $-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin 2x < 1 \xRightarrow{\sin 2x = \frac{m-1}{4}} -\frac{1}{2} < \frac{m-1}{4} < 1$

$\Rightarrow -2 < m-1 < 4 \Rightarrow \boxed{-1 < m < 5}$

3- $\tan x + \cot x = \frac{1}{\sin x \cos x} = -3 \Rightarrow \boxed{\sin x \cos x = -\frac{1}{3}}\ I$ $\quad 3\pi < 4x < 4\pi \Rightarrow \frac{3}{4}\pi < x < \pi \Rightarrow \sin x + \cos x < 0\ (III)$

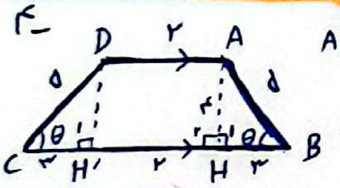
$\sin^2 x + \cos^2 x = 1 \xRightarrow{(I)} \sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{1}{3} \Rightarrow (\sin x + \cos x)^2 = \frac{1}{3} \xRightarrow{(III)} \sin x + \cos x = -\sqrt{\frac{1}{3}}$

$\Rightarrow \sin^3 x + \cos^3 x = (\sin x + \cos x)^3 - 3\sin x\cos x(\sin x + \cos x)$

$= -\frac{1}{3}\sqrt{\frac{1}{3}} - 3\left(-\frac{1}{3}\right)\left(-\sqrt{\frac{1}{3}}\right) = -\frac{1}{3}\sqrt{\frac{1}{3}} - \sqrt{\frac{1}{3}} = -\frac{4}{3}\sqrt{\frac{1}{3}}$

$\Rightarrow \frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{-\frac{4}{3}\sqrt{\frac{1}{3}}} = \frac{1}{-\sqrt{\frac{16}{27}}} = -\frac{\sqrt{27}}{\sqrt{16}} = \boxed{-\frac{3}{4}\sqrt{3}}$

4-



$A\hat{H}B$:

$\cos\theta = \frac{BH}{AB} = \frac{BH}{5} = \frac{6}{10} \Rightarrow BH = 3$

فیثاغورث $\Rightarrow AH^2 + BH^2 = AB^2 \Rightarrow AH^2 + 9 = 25 \Rightarrow AH^2 = 16 \Rightarrow AH = 4$

$\left.\begin{array}{l} AB = DC \\ \hat{H}_1 = \hat{H}'_1 = 90^\circ \\ \hat{C} = \hat{B} = \theta \end{array}\right\}$ وز $\Rightarrow A\hat{B}H \cong D\hat{C}H'$ $\xRightarrow[\text{متناظر}]{\text{تساوی اضلاع}}$ $CH' = BH = 3$

$S = \frac{AD + BC}{2} \times AH = \frac{2+8}{2} \times 4 = \boxed{20}$ $\quad (BC = CH' + H'H + BH)$

$\left.\begin{array}{l} AD \parallel HH' \\ \left.\begin{array}{l} AH \perp BC \\ DH' \perp BC \end{array}\right\} \Rightarrow AH \parallel DH' \\ \hat{A}_2 = 90^\circ \end{array}\right\} \Rightarrow ADH'H$ مستطیل است $\Rightarrow AD = HH' = 2$

5- $\tan(285^\circ)\tan(-165^\circ) - \sin(1095^\circ)\cos(255^\circ) \overset{(I)}{=} \underbrace{-\cot(15^\circ)\tan(15^\circ)}_{-1} - \sin(15^\circ)(-)\sin(15^\circ)$

$\left.\begin{array}{l} \tan(285^\circ) = \tan(270^\circ + 15^\circ) = \tan(\frac{3\pi}{2} + 15^\circ) = -\cot 15^\circ \\ \tan(-165^\circ) = \tan(15^\circ - 180^\circ) = \tan(15^\circ - \pi) = \tan 15^\circ \\ \sin(1095^\circ) = \sin(1080^\circ + 15^\circ) = \sin(6\pi + 15^\circ) = \sin 15^\circ \\ \cos(255^\circ) = \cos(270^\circ - 15^\circ) = \cos(\frac{3\pi}{2} - 15^\circ) = -\sin 15^\circ \end{array}\right\}(I)$

$= -1 + \sin^2(15^\circ) = -(1 - \sin^2 15^\circ)$

$= -\cos^2 15^\circ$

$\Rightarrow -\cos^2 15^\circ = K\cos^2 15^\circ$

$\Rightarrow \boxed{K = -1}$

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6- $A=\sqrt{3}\cos(210^\circ)\underbrace{\sin(243^\circ)}_{-\cos(27^\circ)}-\sqrt{2}\sin(135^\circ)\underbrace{\cos(153^\circ)}_{-\cos(27^\circ)}$

$=\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)(-)\cos(27^\circ)-\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)(-)\cos(27^\circ)$

$=\frac{3}{2}\cos(27^\circ)+\cos(27^\circ)=\frac{5}{2}\cos(27^\circ)$ $\quad \frac{\frac{5}{2}\cos(27^\circ)}{\cos(27^\circ)}=\boxed{\frac{5}{2}}$

7- $f(x)=16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x)$

$\xrightarrow{\times\frac{\sin^2(3x)}{\sin^2(3x)}} f(x)=\frac{16\,\overbrace{(\sin^2(3x)\cos^2(3x))}^{\sin^2(6x)}\cos^2(6x)\cos^2(12x)\cos^2(24x)}{\sin^2(24x)\sin^2(3x)}=\frac{(4\overbrace{\sin^2(6x)\cos^2(6x)}^{\sin^2(12x)})\cos^2(12x)\cos^2(24x)}{\sin^2(3x)}$

$\xrightarrow{\times\frac{16}{16}}\frac{16(4\sin^2(12x)\cos^2(12x))\cos^2(24x)}{16\sin^2(3x)}=\frac{4\sin^2(24x)\cos^2(24x)}{16\sin^2(3x)}=\frac{\sin^2(48x)}{16\sin^2(3x)}$

$\Rightarrow f\left(\frac{\pi}{72}\right)=\frac{\sin^2\left(\frac{4\pi}{6}\right)}{16\sin^2\left(\frac{\pi}{12}\right)}=\frac{\frac{3}{4}}{16\times\left(\frac{2-\sqrt{3}}{4}\right)}=\frac{3}{16(2-\sqrt{3})}\xrightarrow{\times\frac{(2+\sqrt{3})}{(2+\sqrt{3})}}\frac{3(2+\sqrt{3})}{16(4-3)}=\boxed{\frac{6+3\sqrt{3}}{16}}$

$\sin^2\frac{\pi}{12}=\frac{1-\cos\frac{\pi}{6}}{2}=\frac{1-\frac{\sqrt{3}}{2}}{2}=\frac{2-\sqrt{3}}{4}$

8- $1-\sin x=4+4\sin x\Rightarrow 5\sin x=-3\Rightarrow\boxed{\sin x=-\frac{3}{5}}\Rightarrow\sin^2 x=\frac{9}{25}\Rightarrow 1-\sin^2 x=\frac{16}{25}$

$\Rightarrow\cos^2 x=\frac{16}{25}\Rightarrow\cos x=\pm\frac{4}{5}$ $\xrightarrow{\text{در ربع سوم است}}\boxed{\cos x=-\frac{4}{5}}$

$\tan\frac{x}{2}=\frac{\sin x}{1+\cos x}=\frac{-\frac{3}{5}}{1-\frac{4}{5}}=\frac{-\frac{3}{5}}{\frac{1}{5}}=\boxed{-3}$

اثبات: $\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\tan\frac{x}{2}$

9- $\frac{\sin\theta}{1-\cos\theta}=\frac{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}=\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}=\cot\frac{\theta}{2}\quad(I)$

$\frac{\sin\theta}{1-\cos\theta}\xrightarrow{\times\frac{1+\cos\theta}{1+\cos\theta}}\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}=\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}=\frac{1+\cos\theta}{\sin\theta}\overset{(I)}{=}\cot\frac{\theta}{2}$

$\Rightarrow\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\cot\frac{\theta}{2}=k\cot\frac{\theta}{2}\Rightarrow\boxed{k=2}$

10- $\sin\alpha=\frac{\sqrt{2}}{10}\Rightarrow\sin^2\alpha=\frac{2}{100}\Rightarrow 1-\sin^2\alpha=\frac{98}{100}\Rightarrow\cos^2\alpha=\frac{98}{100}\Rightarrow\cos\alpha=\pm\frac{7\sqrt{2}}{10}$

$\xrightarrow{\alpha\ \text{در ربع دوم است}}\boxed{\cos\alpha=-\frac{7\sqrt{2}}{10}}$

$\cos\left(\frac{11\pi}{4}+\alpha\right)=\underbrace{\cos\left(\frac{11\pi}{4}\right)}_{\cos\left(\frac{3\pi}{4}\right)}\cos(\alpha)-\underbrace{\sin\left(\frac{11\pi}{4}\right)}_{\sin\left(\frac{3\pi}{4}\right)}\sin(\alpha)=\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{7\sqrt{2}}{10}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{10}\right)$

$=\frac{14}{20}-\frac{2}{20}=\frac{12}{20}=\boxed{\frac{6}{10}}$