نام و نام خانوادگی ...کارلو... رحیم پور...... پاسخنامه تشریحی تکلیف شماره ...۱۱... کلاس ۲۸ ...... دوشنبه ۱۴۰۲/۰۶/۰۴

از تساوی $\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$ می توان فهمید که $\cot\alpha$ و $\cos\alpha$ هم علامتند (مخرج همواره مثبت است) که در دایره مثلثاتی برابر با ربع اول و ۲ می شود. ۱

(۲)

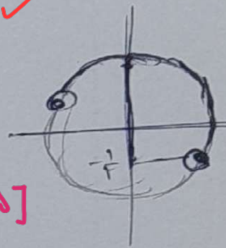
$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$ → ناحیه اول: $\frac{1-\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{\cos\alpha}$ درست ✓

→ و ناحیه دوم: $-\left(\frac{1+\sin\alpha}{\cos\alpha}\right) \neq \frac{\sin\alpha-1}{\cos\alpha}$ نادرست ✗

با امتحان رابطه دیگر در دو ناحیه مشترک اول ناحیه دو محقق است و جواب → ناحیه اول ✓

$-\frac{\pi}{12} < x < \frac{5\pi}{12} \rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \rightarrow -\frac{1}{2} < \sin 2x \leq 1$ ✓ ۲

(۱)



$-\frac{1}{2} < \sin 2x \leq 1 \rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1 \rightarrow m \in (-1, 5]$

$\tan x + \cot x = \frac{1}{\sin x \cos x} = -4 \rightarrow \boxed{\sin x \cos x = -\frac{1}{4}}$ ✓, $(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2(\sin x \cos x)$

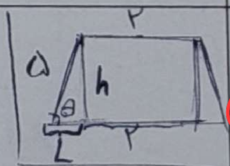
$\rightarrow \boxed{\sin x + \cos x = \sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2}} \rightarrow \sin^3 x + \cos^3 x = ~~(\sin x + \cos x)^3 - 3(\sin x \dots~~$ ۳

(۱)

$= (\sin x + \cos x)^3 - 3(\sin x + \cos x)(\sin x \cos x) = \frac{2\sqrt{2}}{8} - 3\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{4}\right) \rightarrow$

$\rightarrow -\frac{2\sqrt{2}}{9} \rightarrow$ معکوس $\rightarrow \frac{9}{-2\sqrt{2}} = \frac{9\sqrt{2}}{-4}$

$5 \times \cos\theta = L = 5 \times 0.6 = 3 \rightarrow$ طول قاعده پایینی $= 3 + 2 + 3 = 8$



$\sin^2 x + \cos^2 x = 1 \rightarrow \cos\theta = 0.8,\ 5 \times \sin\theta = h \rightarrow h = 5 \times 0.8 = 4$ ۴

(۲)

$S_{\text{ذوزنقه}} = \frac{(2+8) \times 4}{2} = 20$ ✓

$\tan\left(\frac{3\pi}{2} + 10^\circ\right)\tan(\pi + 10^\circ) - \sin(2\pi + 10^\circ) \times \cos\left(\frac{3\pi}{2} - 10^\circ\right) =$

(۲)

$= -\cot(10) \times \tan(10) + \sin(10) \times \sin(10) = -1 + \sin^2(10) \rightarrow$

$\rightarrow = -(1 - \sin^2(10)) = -\cos^2 10 = k\cos^2(10) \rightarrow \boxed{k = -1}$ ✓ ۵

$$\sqrt{3}\left(-\frac{\sqrt{3}}{3}\right)\left(\underbrace{\sin\left(\frac{3\pi}{2}-27^\circ\right)}_{-\cos(27)}-\sqrt{2}\left(+\frac{\sqrt{2}}{2}\right)\underbrace{\left(\cos(\pi-27)\right)}_{-\cos(27)}\right)\rightarrow$$

$$\rightarrow \cos(27)-\cos(27)=0 = \text{عبارت} \Rightarrow \frac{0}{\cos(27)}=0$$

صفر

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$$16\times\cos^2\left(\frac{\pi}{12}\right)\times\cos^2\left(\frac{\pi}{6}\right)\times\cos^2\left(\frac{\pi}{3}\right)\times\cos^2\left(\frac{2\pi}{3}\right)=f(x)$$

$$\boxed{\cos^2\frac{\pi}{12}=\frac{1+\cos\frac{\pi}{6}}{2}=\frac{3}{4}}\rightarrow f(x)=16\times\frac{3}{4}\times\left(\frac{1}{2}\right)^2\times\left(\frac{\sqrt{3}}{2}\right)^2\times\left(-\frac{1}{2}\right)^2\rightarrow$$

$$\rightarrow=\boxed{\frac{9}{16}}$$

۱ | ۷

$$1-\sin x=4+4\sin x\rightarrow\sin x=-\frac{3}{5}=-0.6,\quad \sin^2x+\cos^2x=1\rightarrow\boxed{\cos x=-0.8}$$

ربع سوم!

$$\cos\frac{x}{2}=\sqrt{\frac{1+0.8}{2}}=\sqrt{\frac{9}{10}}=\frac{3}{\sqrt{10}}\rightarrow\sin^2x+\cos^2x=1\rightarrow$$

$$\rightarrow\sin\frac{x}{2}=\frac{1}{\sqrt{10}}\rightarrow\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\frac{1}{\sqrt{10}}}{\frac{3}{\sqrt{10}}}=\boxed{\frac{1}{3}}$$

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$$\text{عبارت}=\frac{\sin^2\theta+1-\cos^2\theta}{\sin\theta(1-\cos\theta)}=\frac{2\sin^2\theta}{\sin\theta(1-\cos\theta)}=\frac{2\sin\theta}{1-\cos\theta}\ \Big|\ K\cot\frac{\theta}{2}=K\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}\rightarrow$$

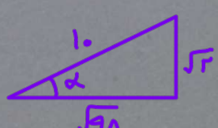
$$=K\frac{\sqrt{\frac{1+\cos\theta}{2}}}{\sqrt{\frac{1-\cos\theta}{2}}}=\frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}}K=\frac{2\sqrt{1-\cos^2\theta}}{1-\cos\theta}=\frac{2\sqrt{1-\cos\theta}\times\sqrt{1+\cos\theta}}{1-\cos\theta}$$

بنا بر رابطه فوق $K=2\times\sqrt{1-\cos\theta}$ ، عبارت غلط خورده از تبدیل $\sin^2x+\cos^2x=1$ بدست آمده

۱ | ۹

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\left(3\pi-\frac{\pi}{4}+\alpha\right)=-\cos\left(\alpha-\frac{\pi}{4}\right)$$

$$=-\left(\cos\alpha\cos\frac{\pi}{4}+\sin\alpha\sin\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha)$$



ناحیه دوم $\rightarrow \cos\alpha=-\frac{\sqrt{98}}{10}$

$$-\frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha)=-\frac{\sqrt{2}}{2}\left(-\frac{\sqrt{98}}{10}+\frac{\sqrt{2}}{10}\right)=\boxed{\frac{3}{5}}$$

۰ | ۱۰

-۳

$(\sin\alpha+\cos\alpha)^2 = 1+2\sin\alpha\cos\alpha$

$= 1+2(\frac{-1}{3}) = \frac{1}{3}$

$3\pi < 4x < 4\pi \rightarrow \frac{3}{4}\pi < x < \pi \xrightarrow{\sin x+\cos x<0} \frac{-\sqrt{3}}{3}$

$\sin^3 x+\cos^3 x = (\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x) = -\frac{\sqrt{3}}{3}(\frac{4}{3})$

$\hookrightarrow 1-(-\frac{1}{3}) = \frac{4}{3}$

$\rightarrow \frac{1}{\sin^3 x+\cos^3 x} = \boxed{\frac{-3}{4}\sqrt{3}}$

-۶

$A = \sqrt{3}(-\frac{\sqrt{3}}{2})\sin(270-27) - \sqrt{2}\times\frac{\sqrt{2}}{2}\cos(180-27)$

$A = \frac{3}{2}\sin 27 + \cos 27 = \frac{5}{2}\cos 27$

-۷

$f(\frac{\pi}{36}) = 16\cos^2(\frac{\pi}{12})\cos^2(\frac{\pi}{6})\cos^2(\frac{\pi}{3})\cos^2(\frac{2\pi}{3})$

$\cos^2\frac{\pi}{12} = \frac{1+\cos\frac{\pi}{6}}{2}$, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\rightarrow \cos^2\frac{\pi}{12} = \frac{2+\sqrt{3}}{4}$

$f(\frac{\pi}{36}) = 16(\frac{2+\sqrt{3}}{4})\times\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4} = \boxed{\frac{3(2+\sqrt{3})}{16}}$

-۸

$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{-3}{5} \rightarrow 10\tan\frac{x}{2} = -3-3\tan^2\frac{x}{2}$

$\tan\frac{x}{2} = \frac{-1}{3}$ × غیرصحیح!

$\boxed{\tan\frac{x}{2} = -3}$ ✓

-۹

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta+(1-\cos^2\theta)}{(1-\cos\theta)(\sin\theta)} = \frac{2\sin^2\theta}{\sin\theta(1-\cos\theta)}$

$\frac{2\sin\theta}{1-\cos\theta} = \frac{2\times2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = 2\cot\frac{\theta}{2} \rightarrow \boxed{K=2}$