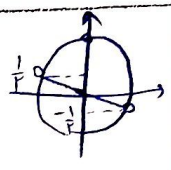


$$\cot \alpha = \frac{\cos \alpha}{\sqrt{1 - \cos^2 \alpha}} = \frac{\cos \alpha}{\sqrt{\sin^2 \alpha}} = \frac{\cos \alpha}{|\sin \alpha|} \rightarrow \sin \alpha > 0 \quad (1)$$

$$\frac{1}{\sqrt{\cos^2 \alpha}} - \frac{1}{\cot \alpha} = \frac{1 - \sin \alpha}{|\cos \alpha|} \rightarrow \frac{1}{|\cos \alpha|} - \frac{\sin \alpha}{\cos \alpha} = \frac{1 - \sin \alpha}{|\cos \alpha|} \rightarrow \frac{1 - 1 + \sin \alpha}{|\cos \alpha|} = \frac{\sin \alpha}{\cos \alpha}$$

$$\rightarrow \cos \alpha > 0 \quad (2) \quad (1), (2) \rightarrow \alpha \text{ در ربع اول و چهارم}$$

$$-\frac{\pi}{12} < \alpha < \frac{\pi}{12} \rightarrow -\frac{\pi}{4} < \alpha < \frac{\pi}{4} \quad \sin \alpha = \frac{m-1}{\varepsilon} \quad -\frac{1}{\varepsilon} < \frac{m-1}{\varepsilon} < 1$$

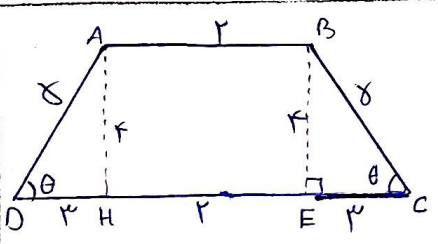


$$\rightarrow -1 < m-1 < \varepsilon \rightarrow -1 < m < \varepsilon + 1$$

$$\tan \alpha + \cot \alpha = \frac{1}{\sin \alpha \cos \alpha} = -2 \rightarrow \sin \alpha \cos \alpha = -\frac{1}{2} \rightarrow (\sin \alpha + \cos \alpha)^2 = 1 + 2 \sin \alpha \cos \alpha$$

$$= \frac{1}{\varepsilon^2} \rightarrow |\sin \alpha + \cos \alpha| = \frac{\sqrt{\varepsilon^2}}{\varepsilon} \quad \frac{\frac{\pi}{4} < \alpha < \pi \rightarrow \sin \alpha + \cos \alpha = -\frac{\sqrt{\varepsilon}}{\varepsilon}$$

$$\rightarrow \frac{1}{\sin \alpha + \cos \alpha} = \frac{1}{(\sin \alpha + \cos \alpha)(\sin^2 \alpha + \cos^2 \alpha - \sin \alpha \cos \alpha)} = \frac{1}{-\frac{\sqrt{\varepsilon}}{\varepsilon} \left(1 + \frac{1}{\varepsilon}\right)} = \frac{-\varepsilon}{\varepsilon \sqrt{\varepsilon}}$$



$$\cos \theta = \frac{EC}{BC} = \frac{EC}{\delta} = \frac{\gamma}{1} \rightarrow EC = \gamma \quad \text{تساوی اضلاع} \quad (BE = \varepsilon)$$

$$\rightarrow \frac{HD}{DA} = \frac{HD}{\delta} = \frac{\gamma}{1} \rightarrow HD = \gamma$$

$$\rightarrow DC = n \quad \rightarrow S = \frac{(AB + DC)(BE)}{2} = \frac{(r + n)(\gamma)}{2}$$

(20)

$$\tan(18^\circ) \tan(-14^\circ) - \sin(109^\circ) \cos(188^\circ) = \tan\left(\frac{\pi}{10} + 18^\circ\right) \tan(-\pi + 18^\circ)$$

$$- \sin(9\pi + 18^\circ) \cos\left(\frac{\pi}{10} - 18^\circ\right) = (-\cos 18^\circ)(\tan 18^\circ) - (\sin 18^\circ)(-\sin 18^\circ)$$

$$= -1 - (-\sin^2 18^\circ) = -1 + \sin^2 18^\circ = -\cos^2 18^\circ \rightarrow k = -1$$

$$\begin{aligned}
 A &= \sqrt{r} \cos(11^\circ) \sin(124^\circ) - \sqrt{r} \sin(11^\circ) \cos(124^\circ) \\
 &= -\frac{r}{r} \sin\left(\frac{12\pi}{r} - 12^\circ\right) - (1) \cos(\pi - 12^\circ) = -\frac{r}{r} (-\cos 12^\circ) - (-\cos 12^\circ) \\
 &= r, \delta \cos 12^\circ \rightarrow \frac{A}{\cos 12^\circ} = \boxed{r, \delta}
 \end{aligned}$$

$$\begin{aligned}
 \cos 18^\circ &= \cos(28^\circ - 10^\circ) = \cos 28^\circ \cos 10^\circ + \sin 28^\circ \sin 10^\circ = \frac{\sqrt{r}}{r} + \frac{\sqrt{r}}{r} = \frac{\sqrt{r} + \sqrt{r}}{r} \\
 \rightarrow 14 \cos^2(18^\circ) \cos^2(10^\circ) \cos^2(4^\circ) \cos^2(12^\circ) &= 14 \left(\frac{\sqrt{r} + \sqrt{r}}{r}\right)^2 \left(\frac{\sqrt{r}}{r}\right)^2 \left(\frac{1}{r}\right)^2 \left(\frac{1}{r}\right)^2 \\
 &= 14 \frac{r + 2\sqrt{r}}{r} \times \frac{r}{r} \times \frac{1}{r} \times \frac{1}{r} = \frac{r + 2\sqrt{r}}{r} = \frac{r + 2\sqrt{r}}{r}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1 - \sin x}{1 + \sin x} = r &\rightarrow 1 - \sin x = r + r \sin x \rightarrow \delta \sin x = -r \rightarrow \sin x = \frac{-r}{\delta} \\
 \xrightarrow{\text{further}} \cos x = \frac{-r}{\delta} &\rightarrow \tan\left(\frac{x}{r}\right) = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x} = \frac{\frac{-r}{\delta}}{\frac{1}{\delta}} = \boxed{-r}
 \end{aligned}$$

$$\frac{\sin \alpha}{1 + \cos \alpha} = \frac{r \sin \frac{\alpha}{r} \cos \frac{\alpha}{r}}{r \cos^2 \frac{\alpha}{r}} = \tan \frac{\alpha}{r} \left\{ \frac{1 - \cos \alpha}{\sin \alpha} = \frac{r \sin^2 \frac{\alpha}{r}}{r \sin \frac{\alpha}{r} \cos \frac{\alpha}{r}} = \tan \frac{\alpha}{r} \right.$$

$$\begin{aligned}
 \rightarrow \frac{\sin \alpha}{1 - \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} &= \cot \frac{\alpha}{r} + \cot \frac{\alpha}{r} = r \cot \frac{\alpha}{r} = k \cot \frac{\alpha}{r} \\
 &\rightarrow \boxed{k = r}
 \end{aligned}$$

$$\begin{aligned}
 \sin \alpha = \frac{\sqrt{r}}{r_0} &\rightarrow \sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \cos^2 \alpha = \frac{r_0 - r}{r_0} \xrightarrow{\text{further}} \cos \alpha = \frac{-\sqrt{r_0 - r}}{r_0} \\
 \rightarrow \cos\left(\frac{11\pi}{2} + \alpha\right) &= \left(\cos \frac{11\pi}{2}\right) (\cos \alpha) - \left(\sin \frac{11\pi}{2}\right) (\sin \alpha) = \left(\frac{-\sqrt{r}}{r}\right) \left(\frac{-\sqrt{r_0 - r}}{r_0}\right) \\
 - \left(\frac{\sqrt{r}}{r}\right) \left(\frac{\sqrt{r_0 - r}}{r_0}\right) &= \frac{r_0 - r}{r_0} - \frac{r}{r_0} = \frac{r_0 - 2r}{r_0}
 \end{aligned}$$