① $\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|}$ → همواره ⊕ ⇒ علامت $\cot\alpha$ و $\cos\alpha$ یکسان است

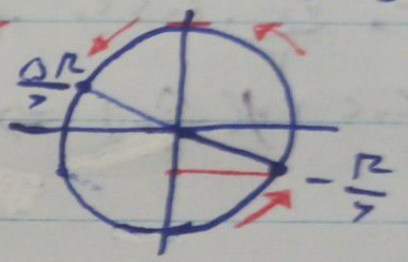
$$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$\cos\alpha > 0$: $\frac{1}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{\cos\alpha}$ ✓

$\cos\alpha < 0$: $\frac{-1}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha} \neq \frac{1-\sin\alpha}{\cos\alpha}$ ✗ غ ق ج

$\Rightarrow \cos\alpha > 0 \Rightarrow \cot\alpha > 0 \Rightarrow \sin\alpha > 0$ ⇒ در ناحیه‌ی اول است

② $-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6}$



$-\frac{1}{2} < \sin 2x \leq 1 \Rightarrow \frac{-1}{2} < \frac{m-1}{2} \leq 1 \Rightarrow -1 < m \leq 5$

③ $\tan x + \cot x = \frac{2}{\sin 2\alpha} = -3 \Rightarrow \frac{\sin 2\alpha}{2} = \frac{-1}{3}$

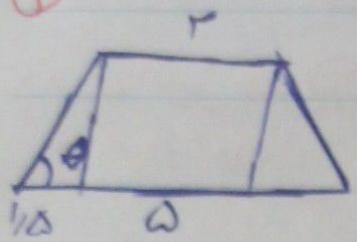
$\sin^3\alpha + \cos^3\alpha = (\sin\alpha + \cos\alpha)(\underbrace{\sin^2\alpha + \cos^2\alpha}_{1} - \underbrace{\sin\alpha\cos\alpha}_{\frac{\sin 2\alpha}{2}})$

$= -\sqrt{1 + \sin 2\alpha}$

$= \frac{\sqrt{3}}{3}$

$$\Rightarrow \frac{\sqrt{3}}{2}\left(\frac{3}{2}\right) = \frac{3\sqrt{3}}{4}$$

۴)



$$\frac{10}{7} \times 1/5 = 2/5$$

$$2/5 \times 0/8 = 0/2 \quad \text{ارتفاع}$$

$$\frac{(2+5)\,0/2}{2} = 0/7$$

۵) $$\tan\left(15 + \frac{3\pi}{2}\right)\tan(15 - \pi) - \sin(15)\cos\left(\frac{3\pi}{2} - 15\right)$$

$$\cot(15)\tan(15) + \sin(15)\sin(15) = 1 + \sin^2(15)$$

$$\frac{1+\sin^2(15)}{2\cos^2(15)} = \frac{6+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} = k$$

۶) $$\sqrt{3}\cos\left(\frac{\pi}{6} + \pi\right)\sin\left(\frac{3\pi}{2} - 2x\right) - \sqrt{2}\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\cos(\pi - 2x)$$

$$\sqrt{3}\cos\left(\frac{\pi}{6}\right)\cos(2x) + \sqrt{2}\cos\left(\frac{\pi}{4}\right)\cos(2x)$$

$$\Rightarrow \sqrt{3}\cdot\frac{\sqrt{3}}{2}\cdot\cos 2x + \sqrt{2}\cdot\frac{\sqrt{2}}{2}\cdot\cos 2x = \cos 2x\left(\frac{3}{2} + 1\right)$$

$$\Rightarrow (\cos 2x)\left(\frac{5}{2}\right) \Rightarrow \frac{5}{2}\cos 2x \quad \text{برابر}$$

۷) $$\frac{\pi}{3x} \Rightarrow 16\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{3}\right)\cos^2\left(\frac{2\pi}{3}\right)$$

$$2\cos^2\left(\frac{\pi}{12}\right) = 1 + \cos\left(\frac{\pi}{6}\right) = 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}$$

$$\Rightarrow 16\left(\frac{2+\sqrt{3}}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{6+3\sqrt{3}}{16}$$

Date Subject

(٨) $1-\sin x = 2+3\sin x \Rightarrow 4\sin x = -2$

$\Rightarrow \sin x = -\frac{2}{4}$ $\cos x = -\frac{4}{5} \Rightarrow \frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$

$\frac{-2}{5}$ / $\frac{1}{5}$ $= -2$ ✓

(٩) $\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2} \Rightarrow \frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\cot\frac{\theta}{2}$

$\Rightarrow K = 2$ ✓

(١٠) $\cos\left(\frac{3\pi}{4}+\alpha\right) = \cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha$

$\Rightarrow \frac{-\sqrt{2}}{2}\left(-\left(\frac{-\sqrt{98}}{10}\right) - \frac{\sqrt{2}}{10}\right) = \frac{\sqrt{2}}{2}\left(\frac{7\sqrt{2}}{10} - \frac{\sqrt{2}}{10}\right)$

$= \frac{7\sqrt{4}}{10} \cdot \frac{\sqrt{2}}{4} = \frac{6}{10}$ ✓