سید مهدی میرعبدالباقی ۲۰ آفرین!

۱- $1-\cos^2\alpha = \sin^2\alpha \Rightarrow \sqrt{\sin^2\alpha} = |\sin\alpha|$ (۲)

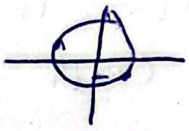
$\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha > 0$ I

$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha = |\cos\alpha|$

$\cos\alpha > 0$ II

I ∩ II ⇒ ناحیه اول ✓

۲- $-\frac{\pi}{4} < \frac{m-1}{4}\pi < \frac{5\pi}{4}$ (۲)



$-\frac{1}{2} < \frac{m-1}{4} \leq 1$

$\Rightarrow -2 < m-1 \leq 4$

$-1 < m \leq 5$ ✓

۳- $\tan\pi + \cot\pi = \frac{1}{\sin\pi\cos\pi} = -3 \quad \sin\pi\cos\pi = -\frac{1}{3}$

$\sin^3\pi + \cos^3\pi = (\sin\pi + \cos\pi)(\sin^2\pi - \sin\pi\cos\pi + \cos^2\pi)$ (۲)

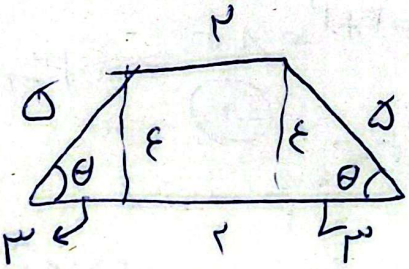
$(\sin\pi+\cos\pi)^2 = 1 + 2\sin\pi\cos\pi = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow |\sin\pi+\cos\pi| = \frac{\sqrt{3}}{3}$

$\Rightarrow \sin\pi+\cos\pi = -\frac{\sqrt{3}}{3}$ (لو کوچکتر از صفر)

$\Rightarrow -\frac{\sqrt{3}}{3}\left(1+\frac{1}{3}\right) = \sin^3\pi+\cos^3\pi = -\frac{4}{9}\sqrt{3}$

$\frac{1}{\sin^3\pi+\cos^3\pi} = \frac{-9}{4\sqrt{3}} = -\frac{3}{4}\sqrt{3}$ ✓

۴- (۲)




$S = 8 + 12 = 20$ ✓

۵- $\tan(\frac{5\pi}{2}+\theta)\times\tan(\pi-\theta)$

$-\cot(\theta)\times\tan(\theta)=-1$

$\sin(4\pi+\theta)\times\cos(\frac{3\pi}{2}-\theta)$

$-\sin(\theta)$

$\Rightarrow -1+\sin^2(\theta)$

$-(1-\sin^2(\theta))$

$-\cos^2\theta$

$-1<-1$ ✓ ۵

۶- $\sqrt{2}\sin\frac{\pi}{4}\times\sin(\frac{3\pi}{2}-\theta)-\sqrt{2}\times\frac{\sqrt{2}}{2}\times\cos(\pi-\theta)$

$-\cos\theta$ $-\cos\theta$

$\frac{3}{2}\cos\theta+\cos\theta=\frac{5}{2}\cos\theta$

$\frac{\frac{5}{2}\cos\theta}{\cos\theta}=\frac{5}{2}$ ✓ ۲

۷- $14\times\cos^2(\frac{\pi}{12})\times\cos^2(\frac{\pi}{4})\times\cos^2(\frac{\pi}{12})\times\cos^2(\frac{3}{12}\pi)$

$14\times\frac{2+\sqrt{3}}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{2+\sqrt{3}}{14}$ ✓

$\frac{1+\cos\frac{\pi}{6}}{2}=\cos^2\frac{\pi}{12}=\frac{2+\sqrt{3}}{4}$ ۲

۸- $\tan\pi=\frac{2\tan\frac{\pi}{2}}{1-\tan^2\frac{\pi}{2}}$ $14\cos\theta\sin\theta=1-\sin\theta$

$\frac{3}{4}=\frac{2\tan\frac{\pi}{2}}{1-\tan^2\frac{\pi}{2}}$ $\tan\frac{\pi}{2}=t$ $\Rightarrow$ $8\sin\theta=-2\Rightarrow\sin\theta=-\frac{2}{5}$

$\Rightarrow\cos\theta=-\frac{\sqrt{21}}{5}$

$3-3t^2=4t$ $\Rightarrow 3t^2+4t-3=0$

$t=-3$ ✓ $t=\frac{1}{3}$ غ ق ق ۲

۹- $\frac{2\sin^2\pi}{(1-\cos\pi)\sin\pi}=\frac{2\sin\pi}{1-\cos\pi}\Rightarrow\frac{2\sin\pi}{2\sin^2\frac{\pi}{2}}=\frac{2\sin\frac{\pi}{2}\cos\frac{\pi}{2}}{\sin^2\frac{\pi}{2}}$ ۲

$\Rightarrow 2\frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}}\Rightarrow 2\cot\frac{\pi}{2}$ ✓ $k$

$\frac{1-\cos\pi}{2}=\sin^2\frac{\pi}{2}$ $\sin\pi=2\sin\frac{\pi}{2}\cos\frac{\pi}{2}$

١- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ (٢)

$\cos\alpha$ ، $\sin\alpha$ .  $1-\frac{٢}{١-} = \frac{٩٨}{١٠٠} = \cos^{٢}\alpha = \cos\alpha = \frac{-٧\sqrt{٢}}{١٠}$

$-\frac{\sqrt{٢}}{٢}\alpha - \frac{٧\sqrt{٢}}{١٠} - \frac{\sqrt{٢}}{١٠}\alpha\frac{\sqrt{٢}}{٢} = \frac{٧}{١٠} - \frac{١}{١٠} = \frac{٦}{١٠}$ ✓