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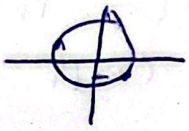
۱- $1-\cos^2\alpha = \sin^2\alpha$ $\quad \sqrt{\sin^2\alpha} = |\sin\alpha|$

$\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha > 0$ I

$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha = |\cos\alpha|$

$\cos\alpha > 0$ II

I ∩ II ⇒ ناحیه اول

۲- $-\frac{\pi}{4} < 2m < \frac{5\pi}{4}$ 

$-\frac{1}{2} < \frac{m-1}{3} \leq 1$

$\Rightarrow -2 < m-1 \leq 3$

$-1 < m \leq 4$

۳-

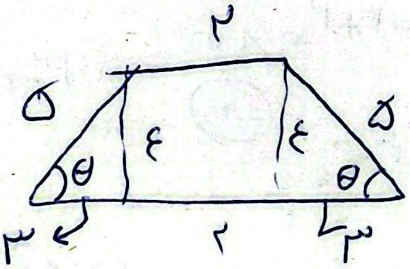
$\tan\pi + \cot\pi = \frac{1}{\sin\pi\cos\pi} = -3$ $\quad \sin\pi\cos\pi = -\frac{1}{3}$

$\sin^3\pi + \cos^3\pi = (\sin\pi + \cos\pi)(\sin^2\pi - \sin\pi\cos\pi + \cos^2\pi)$

$(\sin\pi+\cos\pi)^2 = |\quad + 2\sin\pi\cos\pi = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow |\sin\pi+\cos\pi| = \frac{\sqrt{3}}{3}$ ⇒ $\sin\pi+\cos\pi$ له کوچکتر از صفر $= -\frac{\sqrt{3}}{3}$

$\Rightarrow -\frac{\sqrt{3}}{3}(1+\frac{1}{3}) = \sin^3\pi + \cos^3\pi = -\frac{4}{9}\sqrt{3}$

$\frac{1}{\sin^3\pi+\cos^3\pi} = \frac{-9}{4\sqrt{3}} = -\frac{3}{4}\sqrt{3}$

۴- 

$S = 8 + 12 = 20$

۵-

$$\tan\left(\frac{3\pi}{2}+15\right) \times -\left(\tan(\pi-15)\right)$$
$$-\cot(15) \times \tan(15) = -1$$

$$\sin(4\pi+15) \times \cos\left(\frac{3\pi}{2}-15\right)$$
$$-\sin^2(15)$$

$$\Rightarrow -1+\sin^2(15) = -(1-\sin^2(15)) = -\cos^2 15$$

$$k = -1$$

۶-

$$\sqrt{3}x - \frac{\sqrt{3}}{2} \times \sin\left(\frac{3\pi}{2}-27\right) - \sqrt{2} \times \frac{\sqrt{2}}{2} \times \cos(\pi-27)$$
$$-\cos 27 \qquad -\cos 27$$

$$\frac{3}{2}\cos 27 + \cos 27 = \frac{5}{2}\cos 27$$

$$\frac{\frac{5}{2}\cos 27}{\cos 27} = \frac{5}{2}$$

۷-

$$16 \times \cos^2\left(\frac{\pi}{12}\right) \times \cos^2\left(\frac{\pi}{4}\right) \cos^2\left(\frac{\pi}{3}\right) \times \cos^2\left(\frac{5}{12}\pi\right)$$

$$16 \times \frac{2+\sqrt{3}}{4} \times \frac{3}{4} \times \frac{1}{4} \cdot \frac{1}{4} = \frac{6+3\sqrt{3}}{16}$$

$$\frac{1+\cos\frac{\pi}{6}}{2} = \cos^2\frac{\pi}{12} = \frac{2+\sqrt{3}}{4}$$

۸-

$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

$$\frac{3}{4} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \qquad \tan\frac{x}{2} = t \Rightarrow 3-3t^2 = 8t$$

$$\Rightarrow 3t^2+8t-3=0$$

$$t \to -3, \quad t \to \frac{1}{3} \to \text{غ ق ق}$$

$$16\sin x - 1 - \sin x$$
$$5\sin x = -3 \Rightarrow \sin x = -\frac{3}{5}$$
$$\Rightarrow \cos x = -\frac{4}{5}$$

۹-

$$\frac{2\sin^2 x}{(1-\cos x)\sin x} = \frac{2\sin x}{1-\cos x} \Rightarrow \frac{4\sin x}{4\sin^2\frac{x}{2}} = \frac{4\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}}$$

$$\Rightarrow 2\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = 2\cot \quad k = 2$$

$$\frac{1-\cos x}{2} = \sin^2\frac{x}{2} \qquad \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

10- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos\alpha < 0 \quad \sin\alpha > 0 \qquad 1 - \frac{2}{100} = \frac{98}{100} = \cos^2\alpha \Rightarrow \cos\alpha = \frac{-7\sqrt{2}}{10}$

$= \frac{\sqrt{2}}{2} \times \frac{-7\sqrt{2}}{10} - \frac{\sqrt{2}}{10} \times \frac{\sqrt{2}}{2} = \frac{7}{10} - \frac{1}{10} = \boxed{\frac{6}{10}}$