نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ۲۸ ..... کلاس ..... A

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$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \rightarrow \sin\alpha > 0$

ناحیه اول

$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \rightarrow \frac{1}{|\cos\alpha|} - \frac{1-\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha}$

$\frac{\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \rightarrow \cos\alpha > 0$

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$\sin 2x = \frac{m-1}{4}, \quad -\frac{\pi}{12} < x < \frac{5\pi}{12}$

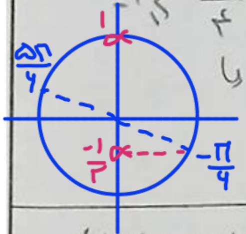
$-\frac{\pi}{6} < 2x < \frac{5\pi}{6} \rightarrow -\frac{1}{2} \leq \sin 2x \leq 1$

$-1 \leq \frac{m-1}{4} \leq 0 \rightarrow \frac{m-1}{4} \leq 0 \rightarrow m \leq 1$

یا

$\frac{m-1}{4} \geq -1 \rightarrow m-1 \geq -4 \rightarrow m \geq -3$

$-3 \leq m \leq 1$



$-\frac{1}{2} < \sin 2x \leq 1 \rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1 \rightarrow m \in (-1, 5]$

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$\tan x + \cot x = -3 \rightarrow \frac{1}{\sin x\cos x} = -3$

$\sin x\cos x = -\frac{1}{3}$

$\sin x + \cos x = -\frac{1}{\sqrt{3}}$

$(\sin x+\cos x)^2 = \sin^2 x+\cos^2 x+2\sin x\cos x = 1-\frac{2}{3} = \frac{1}{3}$

$\frac{1}{\sin^3 x+\cos^3 x} = \frac{1}{(\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x)} = \frac{1}{\frac{-1}{\sqrt{3}} \times \frac{4}{3}} = \frac{-3\sqrt{3}}{4}$

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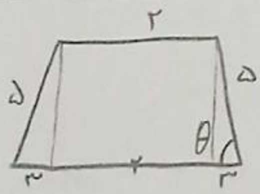

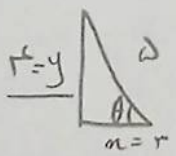

$\cos\theta = \frac{6}{10} = \frac{x}{5} \rightarrow x = 3$

$S_{\text{ذوزنقه}} = \frac{(\text{قاعده بزرگ} + \text{قاعده کوچک}) \times \text{ارتفاع}}{2} = \frac{(2+8)\times 4}{2} = 20$

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$\tan(285°)\tan(-165°) - \sin(1095°)\cos(255°)$

$\tan(270°+15)\tan(-180+15) - \sin(1080+15)\cos(270-15)$

$-\cot(15)\tan(15) - \sin(15)(-\sin(15)) = -1+\sin^2(15) \Rightarrow$

$-\sin^2(15°)-\cos^2(15°)+\sin^2(15°) = -\cos^2(15°) \rightarrow -\cos^2(15°) = k\cos^2 15$

$k = -1$

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$$A = \sqrt{2} \times -\frac{\sqrt{2}}{2} \times \sin\left(\frac{3\pi}{2} - 2\alpha\right) - \sqrt{2} \times \frac{\sqrt{2}}{2} \times \cos(\pi - 2\alpha)$$

$$A = +\frac{2}{2}\left(\cos(2\alpha) + \cos(2\alpha)\right) = \frac{5}{2}\cos(2\alpha)$$

$$\frac{5}{2}\frac{\cos(2\alpha)}{\cos(2\alpha)} = \frac{5}{2}$$ ✓

$$f\left(\frac{\pi}{24}\right) = 14\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{3\pi}{8}\right)$$

$$14\left(\frac{1+\cos 30}{2}\right)\left(\frac{1+\cos 45}{2}\right)\left(\frac{1+\cos 60}{2}\right)\left(\frac{1+\cos 75}{2}\right) = \frac{4+2\sqrt{3}}{2} \times \frac{3}{4} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{4+2\sqrt{3}}{16}$$ ✓

$$\frac{1-\sin\alpha}{1+\sin\alpha} = 2 \rightarrow 1+2\sin\alpha = 1-\sin\alpha \Rightarrow 3\sin\alpha = -2 \rightarrow \sin\alpha = -\frac{2}{3}$$

$$\cos\alpha = -\frac{\sqrt{5}}{3}$$

$$\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{-\frac{2}{3}}{\frac{1}{3}} = -2$$ ✓

$$\cot\frac{\theta}{2} = \frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$$

$$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2} + \cot\frac{\theta}{2} = 2\cot\frac{\theta}{2} = k\cot\frac{\theta}{2}$$

$k = 2$ ✓

$$\cos\left(\frac{11\pi}{2} + \alpha\right) = \cos\frac{11\pi}{2}\cos\alpha - \sin\frac{11\pi}{2}\sin\alpha$$

$\sin\alpha = \frac{\sqrt{6}}{10}$, $\cos\alpha = -\frac{\sqrt{94}}{10}$

$\cos\left(\frac{11\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) = -\frac{\sqrt{2}}{2}$, $\sin\left(\frac{11\pi}{2}\right) = \frac{\sqrt{2}}{2}$

$$-\frac{\sqrt{2}}{2} \times \frac{-\sqrt{94}}{10} - \left(\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{10}\right) = \frac{+\sqrt{188}}{20} - \left(\frac{2}{20}\right) = \frac{+14}{20} - \frac{2}{20} = \frac{12}{20}$$

$$= \frac{3}{5}$$ ✓