امیر صفری تکلیف شماره ۲۸ ۱۱ تیر A

۱ـ

$$\frac{1}{|\cos\alpha|}-\frac{1-\sin\alpha}{|\cos\alpha|}=\tan\alpha$$

$$\frac{\sin\alpha}{|\cos\alpha|}=\tan\alpha \Rightarrow \sin\alpha=\frac{\sin\alpha}{\cos\alpha}\times|\cos\alpha|$$

$$\cos\alpha=|\cos\alpha| \Rightarrow \cos\alpha>0$$ ناحیه اول و چهارم

$$\cot\alpha=\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$$

$$\cot\alpha\times|\sin\alpha|=\cos\alpha$$

$$\frac{\cos\alpha}{\sin\alpha}\times|\sin\alpha|=\cos\alpha$$

$$\Big\}\ \sin\alpha=|\sin\alpha|,\quad \sin\alpha>0$$ ناحیه اول و دوم

$\mathrm{I}\cap\mathrm{II}\Rightarrow$ $\boxed{\text{ناحیه اول}}$

۲ـ

$$-\frac{\pi}{12}<x<\frac{5\pi}{12} \Rightarrow -\frac{\pi}{6}<2x\leq\frac{5\pi}{6}$$

$$-\frac{1}{2}<\sin 2x\leq 1,\qquad -\frac{1}{2}<\frac{m-1}{4}\leq 1$$

$$-2<m-1\leq 4 \Rightarrow \boxed{-1<m\leq 5}$$

۳ـ

$$\tan x+\cot x=-3$$

$$\frac{2}{\sin 2x}=-3 \qquad \sin 2x=\frac{-2}{3} \qquad 2\sin x\cos x=\frac{-2}{3}$$

$$\sin x\cos x=\frac{-1}{3} \quad (1)$$

$$(\sin x+\cos x)^2=1+2\sin x\cos x=1-\frac{2}{3}=\frac{1}{3} \quad (2)$$

$$\sin x+\cos x=\pm\frac{\sqrt{3}}{3}$$

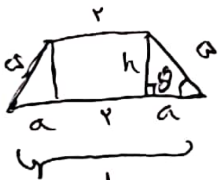
$$\frac{3\pi}{4}<x<\pi \Rightarrow \cos<0,\quad \cos>\sin$$

$$\sin x+\cos x=\frac{-\sqrt{3}}{3} \quad (3)$$

$$\frac{1}{\sin^3x+\cos^3x}=\frac{1}{\underbrace{(\sin x+\cos x)}_{-\frac{\sqrt{3}}{3}}(\underbrace{\cos^2x+\sin^2x}_{1}\underbrace{-\sin\cos}_{-(-\frac{1}{3})})}=$$

$$=\frac{1}{-\frac{4}{3\sqrt{3}}}=\boxed{\frac{-3\sqrt{3}}{4}}$$

۴ـ



$$\cos\theta=\frac{a}{5}$$

$$0.6=\frac{a}{5}\Rightarrow a=3$$

$$\sin^2\theta+\cancel{\cos^2\theta}=1,\quad \cos^2\theta=0.36$$

$$\sin\theta=0.8$$

$$0.8=\frac{h}{5}\Rightarrow h=4$$

$$b=2a+2=8$$

$$S_{\text{ذوزنقه}}=\frac{(2+8)\cancel{4}^{2}}{\cancel{2}}=20$$

۵ـ

$$\tan(285^\circ)\tan(-165^\circ)-\sin(1095^\circ)\cos(255^\circ)=k\cos^2 15^\circ$$

$$\frac{\sin(\frac{3\pi}{2}+15^\circ)}{\cos(\frac{3\pi}{2}+15^\circ)}\times\frac{\sin(\pi+15^\circ)}{\cos(\pi+15^\circ)}-\sin(6\pi+15^\circ)\times\cos(\frac{3\pi}{2}-15^\circ)=k\cos^2 15^\circ$$

$$\frac{-\cancel{\cos 15^\circ}}{\cancel{\sin 15^\circ}}\times\frac{\cancel{-}\cancel{\sin 15^\circ}}{\cancel{-}\cancel{\cos 15^\circ}}-\sin 15^\circ\times(-\sin 15^\circ)=k\cos^2 15^\circ$$

$$-1-(-\sin^2 15^\circ)=\sin^2 15^\circ-1$$

$$\left.\begin{array}{l}\sin^2 15^\circ-1=k\cos^2 15^\circ\\ -\cos^2 15^\circ=k\cos^2 15^\circ\end{array}\right\}\Rightarrow \boxed{k=-1}$$

۶-

$\sqrt{3}\cos(210^\circ)\sin(243^\circ) - \sqrt{2}\sin(135^\circ)\cos(153^\circ) = x\cos 27^\circ$

$\sqrt{3}\cos(\pi+30^\circ)\sin(\frac{3\pi}{2}-27^\circ) - \sqrt{2}\sin(45^\circ)\cos(\pi-27^\circ) = x\cos 27^\circ$

$\sqrt{3}\times -\frac{\sqrt{3}}{2}\times(-\cos 27^\circ)+\sqrt{2}\times\frac{\sqrt{2}}{2}\times(+\cos 27^\circ) = x\cos 27^\circ$

$\frac{3}{2}\cos 27^\circ+\frac{2}{2}\cos 27^\circ = x\cos 27^\circ \Rightarrow \boxed{x=\frac{5}{2}}$

۷-

$f(x) = 16\cos^2 3x\cos^2 6x\cos^2 12x\cos^2 24x$

$f(\frac{\pi}{36}) = 16\times\cos^2 15^\circ\times\cos^2 30^\circ\times\cos^2 60^\circ\times\cancel{\cos^2 120^\circ}$ $\cos^2 60$

$\frac{1+\cos(2\times 15)}{2} = \frac{\sqrt{3}+2}{4}$

$(\frac{\sqrt{3}+2}{4})\times(\frac{\sqrt{3}}{2})^2\times(\frac{1}{2})^2\times(-\frac{1}{2})^2\times 16 =$

$\frac{6+2\sqrt{3}}{\cancel{256}_{16}}\times\cancel{16} = \boxed{\frac{6+2\sqrt{3}}{16}}$

۸-

ربع ۳ x

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

$\frac{1-\sin x}{1+\sin x} = 4$

$4+4\sin x = 1-\sin x$

$\sin x = -\frac{3}{5}$

$\sin^2 x+\cos^2 x = 1$

$\cos x = \pm\frac{4}{5} \xrightarrow[\text{ناحیه ۳}]{\cos<0} \cos x = -\frac{4}{5}$

$\tan\frac{x}{2} = \frac{-\frac{3}{5}}{\frac{1}{5}} = \frac{\frac{9}{5}}{-\frac{3}{5}} = \boxed{-3}$

۹-

می‌دانیم $\frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$

$\frac{\sin\alpha}{1-\cos\alpha}+\frac{1+\cos\alpha}{\sin\alpha} = k\cot\frac{\alpha}{2}$

دو طرف را معکوس می‌کنیم $\longrightarrow$ $\frac{1-\cos\alpha}{\sin\alpha}+\frac{\sin\alpha}{1+\cos\alpha} = k\tan\frac{\alpha}{2}$

$\tan\frac{\alpha}{2}+\tan\frac{\alpha}{2} = k\tan\frac{\alpha}{2}$

$\boxed{k=2}$

۱۰-

α ربع دوم

$\sin\alpha = \frac{\sqrt{2}}{10}$

$\sin^2\alpha+\cos^2\alpha = 1$

$\cos\alpha = \pm\frac{7\sqrt{2}}{10} \xrightarrow[\cos<0]{\text{ربع دوم}} \cos\alpha = -\frac{7\sqrt{2}}{10}$

$\cos(\frac{11\pi}{4}+\alpha) = \cos(2\pi+\frac{3\pi}{4}+\alpha) = \cos(\frac{3\pi}{4}+\alpha) = \cos(\frac{3\pi}{4})\cos\alpha - \sin(\frac{3\pi}{4})\sin\alpha$

$(-\frac{\sqrt{2}}{2}\times -\frac{7\sqrt{2}}{10}) - (\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}) = \frac{7}{10}-\frac{1}{10} = \boxed{0/6}$