۱۷,۵

ماهان صاحبقرانی

① ۲

$$\lim_{x\to 1}\frac{4x^2-7x+3}{5x^2-8x+3}=\lim_{x\to 1}\frac{4(x-1)(x-\frac{3}{4})}{5(x-1)(x-\frac{3}{5})}$$

$$\xrightarrow[\text{رفع ابهام}]{=\frac{0}{0}}\frac{4(x-\frac{3}{4})}{5(x-\frac{3}{5})}\xrightarrow{x=1}\frac{4(\frac{1}{4})}{5(\frac{2}{5})}=\frac{1}{2}\checkmark$$

② ۲

$$\lim_{x\to 0}\frac{|3x-1|-|3x+1|}{x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\begin{array}{l}|3x-1|\xrightarrow{x=0}=1-3x\\ |3x+1|\xrightarrow{x=0}=3x+1\end{array}$$

$$\longrightarrow=\frac{\cancel{1}-3x-3x-\cancel{1}}{x}=\frac{-6x}{x}=-6\checkmark$$

③ ۲

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}=\frac{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)}{\cancel{\sqrt{x}-2}}=4\checkmark$$

④ ۱,۷۵

$$\lim_{x\to 2}\frac{x-\sqrt{2x}}{2x^2-x-6}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{x+\sqrt{2x}}{x+\sqrt{2x}}=\frac{x^2-2x}{(x+\sqrt{2x})(2x^2-x-6)}$$

$$=\frac{x\cancel{(x-2)}}{(x+\sqrt{2x})(2)\cancel{(x-2)}(x+\frac{3}{2})}=\frac{2}{(4)(2)(\frac{7}{2})}=\frac{\cancel{1}}{2}\quad\boxed{\frac{1}{14}}$$

⑤ ۲

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{1+\sqrt{x}}{1+\sqrt{x}}\times\frac{2+\sqrt{5-x}}{2+\sqrt{5-x}}$$

$$\longrightarrow=\frac{\overset{-1}{\cancel{(1-x)}}(2+\sqrt{5-x})}{\cancel{(4-5+x)}(1+\sqrt{x})}=-\frac{4}{2}=-2\checkmark$$

⑥ ۲

$$\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}\times\frac{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}$$

$$\longrightarrow=\frac{\overset{3x-12}{\cancel{(3x+4-16)}}(\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7})}{\underset{5x-20}{\cancel{(5x+7-27)}}(\sqrt{3x+4}+4)}=\frac{3}{5}\times\frac{27}{8}=\frac{81}{40}\checkmark$$

⑦ ۱,۷۵

$$\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{\sqrt[3]{x^2}+1+\sqrt[3]{x}}{\sqrt[3]{x^2}+1+\sqrt[3]{x}}\times\frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2}$$

$$=\frac{(3x+\sqrt{x}-4)\cancel{(\sqrt[3]{x^2}+1+\sqrt[3]{x})}^{\,3}}{(x-1)\cancel{(\sqrt{3x+\sqrt{x}}+2)}\,4}=\frac{3}{4}\times\frac{3\cancel{(\sqrt{x}-1)}(\sqrt{x}+\frac{4}{3})}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)}=\frac{3}{4}\times\frac{7}{\cancel{2}}=\cancel{\frac{7}{8}}\quad\boxed{\frac{21}{8}}$$

۸

۰

$$\lim_{x\to\pi}\frac{1+\cos^{۳}x}{\sin^{۲}x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\begin{array}{l}\cos^{۳}x=-\cos^{۳}(\pi-x)\\ \sin^{۲}x=\sin^{۲}(\pi-x)\end{array}$$

$$\longrightarrow=\frac{1-\cos^{\boxed{۳}}(\pi-x)}{\sin^{۲}(\pi-x)}\xrightarrow{\text{هم‌ارزی}}\frac{1-\left(1-\frac{(\pi-x)^{۲}}{۲}\right)^{۳}}{(\pi-x)^{۲}}$$

$$=\frac{\cancel{1}-\cancel{1}+\frac{\cancel{(\pi-x)^{۲}}}{۲}}{\cancel{(\pi-x)^{۲}}}=\frac{1}{۲}$$

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۹ ۲

$$\lim_{x\to\frac{\pi}{۴}}\frac{1-\tan x}{\sin x-\cos x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\frac{\frac{\overset{-1}{\cancel{\cos x-\sin x}}}{\cos x}}{\cancel{\sin x-\cos x}}$$

$$\longrightarrow=\frac{-1}{\cos x}=-\sqrt{۲}\ \checkmark$$

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۱۰ ۳

$$\lim_{x\to\frac{۳\pi}{۴}}\frac{\tan^{۲}x-1}{\cos ۲x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\frac{\frac{\cancel{\sin^{۲}x-\cos^{۲}x}^{\,-1}}{\cos^{۲}x}}{\cancel{\cos^{۲}x-\sin^{۲}x}}$$

$$=\frac{-1}{\cos^{۲}x}=\frac{-1}{\frac{1}{۲}}=-۲\ \checkmark$$

-۶

$$\xrightarrow{Hop} \lim_{x \to ۲} \frac{۱ - \frac{۱}{\sqrt{۲x}}}{۴x - ۱} = \frac{\frac{۱}{۲}}{۷} = \frac{۱}{۱۴}$$

-۷

$$\xrightarrow{Hop} \lim_{x \to ۱} \frac{\frac{۳ + \frac{۱}{۲\sqrt{x}}}{۲\sqrt{۳x + \sqrt{x}}}}{\frac{۱}{۳\sqrt[۳]{x^۲}}} = \frac{\frac{\frac{۷}{۲}}{۴}}{\frac{۱}{۳}} = \frac{۲۱}{۸}$$

-۸

$$\lim_{x \to \pi} \frac{\cancel{(۱ + \cos x)}(۱ + \cos^۲ x - \cos x)}{\cancel{۱ - \cos^۲ x} \quad ۱ - \cos x} = \boxed{\frac{۳}{۲}}$$