ماهان صاحبقرانی

1)

$$\lim_{x\to 1}\frac{4x^2-7x+3}{5x^2-8x+3}=\lim_{x\to 1}\frac{4(x-1)(x-\frac{3}{4})}{5(x-1)(x-\frac{3}{5})}$$

$$\xrightarrow[\text{رفع ابهام}]{=\frac{0}{0}}\frac{4(x-\frac{3}{4})}{5(x-\frac{3}{5})}\xrightarrow{x=1}\frac{4(\frac{1}{4})}{5(\frac{2}{5})}=\frac{1}{2}$$

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2)

$$\lim_{x\to 0}\frac{|3x-1|-|3x+1|}{x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\begin{array}{l}|3x-1|\xrightarrow{x\approx 0}=1-3x\\ |3x+1|\xrightarrow{x\approx 0}=3x+1\end{array}$$

$$\longrightarrow=\frac{\cancel{1}-3x-3x-\cancel{1}}{x}=\frac{-6x}{x}=-6$$

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3)

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}=\frac{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)}{\cancel{\sqrt{x}-2}}=4$$

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4)

$$\lim_{x\to 2}\frac{x-\sqrt{2x}}{2x^2-x-6}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{x+\sqrt{2x}}{x+\sqrt{2x}}=\frac{x^2-2x}{(x+\sqrt{2x})(2x^2-x-6)}$$

$$=\frac{x\cancel{(x-2)}}{(x+\sqrt{2x})(2)\cancel{(x-2)}(x-\frac{3}{2})}=\frac{2}{(4)(2)(\frac{1}{2})}=\frac{1}{2}$$

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5)

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{1+\sqrt{x}}{1+\sqrt{x}}\times\frac{2+\sqrt{5-x}}{2+\sqrt{5-x}}$$

$$\longrightarrow=\frac{\overset{-1}{\cancel{(1-x)}}(2+\sqrt{5-x})}{\cancel{(4-5+x)}(1+\sqrt{x})}=-\frac{4}{2}=-2$$

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6)

$$\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}\times\frac{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}$$

$$\longrightarrow=\frac{\overset{3x-12}{\cancel{(3x+4-16)}}(\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7})}{\underset{5x-20}{\cancel{(5x+7-27)}}(\sqrt{3x+4}+4)}=\frac{3}{5}\times\frac{27}{8}=\frac{81}{40}$$

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7)

$$\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{\sqrt[3]{x^2}+1+\sqrt[3]{x}}{\sqrt[3]{x^2}+1+\sqrt[3]{x}}\times\frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2}$$

$$=\frac{(3x+\sqrt{x}-4)\overset{3}{\cancel{(\sqrt[3]{x^2}+1+\sqrt[3]{x})}}}{(x-1)\underset{4}{\cancel{(\sqrt{3x+\sqrt{x}}+2)}}}=\frac{3}{4}\times\frac{3\cancel{(\sqrt{x}-1)}(\sqrt{x}+\frac{4}{3})}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)}=\frac{3}{4}\times\frac{\frac{7}{3}}{2}=\frac{7}{8}$$

(۸)

$$\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\begin{array}{l}\cos^3 x=-\cos^3(\pi-x)\\ \sin^2 x=\sin^2(\pi-x)\end{array}$$

$$\longrightarrow=\frac{1-\cos^3(\pi-x)}{\sin^2(\pi-x)}\xrightarrow{\text{هم‌ارزی}}\frac{1-\left(1-\frac{(\pi-x)^2}{2}\right)}{(\pi-x)^2}$$

$$=\frac{\cancel{1}-\cancel{1}+\frac{\cancel{(\pi-x)^2}}{2}}{\cancel{(\pi-x)^2}}=\frac{1}{2}$$

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(۹)

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\frac{\frac{\overset{-1}{\cancel{\cos x-\sin x}}}{\cos x}}{\cancel{\sin x-\cos x}}$$

$$\longrightarrow=\frac{-1}{\cos x}=-\sqrt{2}$$

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(۱۰)

$$\lim_{x\to\frac{3\pi}{4}}\frac{\tan^2 x-1}{\cos 2x}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\frac{\frac{\overset{-1}{\cancel{\sin^2 x-\cos^2 x}}}{\cos^2 x}}{\cancel{\cos^2 x-\sin^2 x}}$$

$$=\frac{-1}{\cos^2 x}=\frac{-1}{\frac{1}{2}}=-2$$