(1)

$$\lim_{x\to 1}\frac{4x^2-7x+3}{2x^2-5x+3}=\lim_{x\to 1}\frac{4(x-1)(x-\frac{3}{4})}{2(x-1)(x-\frac{3}{2})}=\frac{1}{2}$$

قسمت سخت تمرین: ⇐ رفع ابهام میکنیم.

(2)

$$\lim_{x\to 0}\frac{|2x-1|-|2x+1|}{x}\quad x\to 0^+:\ \frac{1-2x-2x-1}{x}=\frac{-4x}{x}=-4$$

$$x\to 0^-:\ \frac{1-2x-2x-1}{x}=\frac{-4x}{x}=-4$$

حاصل حد -4 = (دازم هم نبود دوستانه کردم!)

(3)

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\lim_{x\to 4}\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2}=4$$

(4)

$$\lim_{x\to 2}\frac{x-\sqrt{2x}}{2x^2-x-6}=\lim_{x\to 2}\frac{x-\sqrt{2x}}{(x-2)(2x+3)}\times\frac{x+\sqrt{2x}}{x+\sqrt{2x}}=\frac{x^2-2x}{(x-2)(2x+3)(x+\sqrt{2x})}=\frac{x(x-2)}{(x-2)(2x+3)}\times\frac{1}{x+\sqrt{2x}}$$

$$=\frac{2}{7}\times\frac{1}{4}=\frac{1}{14}$$

(5)

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}=\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}\times\frac{1+\sqrt{x}}{1+\sqrt{x}}\times\frac{2+\sqrt{5-x}}{2+\sqrt{5-x}}=\lim_{x\to 1}\frac{1-x}{4-5+x}\times\frac{2+\sqrt{5-x}}{1+\sqrt{x}}$$

$$=-1\times\frac{4}{2}=-2$$

(6)

$$\lim_{x\to 3}\frac{\sqrt{2x+3}-3}{\sqrt[3]{3x+7}-2}=\lim_{x\to 3}\frac{\sqrt{2x+3}-3}{\sqrt[3]{3x+7}-2}\times\frac{\sqrt{2x+3}+3}{\sqrt{2x+3}+3}\times\frac{\sqrt[3]{(3x+7)^2}+2\sqrt[3]{3x+7}+4}{\sqrt[3]{(3x+7)^2}+2\sqrt[3]{3x+7}+4}$$

$$=\frac{2x-6}{3x-8}\times\frac{\sqrt[3]{(3x+7)^2}+2\sqrt[3]{3x+7}+4}{\sqrt{2x+3}+3}=\frac{2(x-3)}{3(x-3)}\times\frac{28}{8}=\frac{14}{12}$$

(7)

$$\lim_{x\to 1}\frac{\sqrt{2x+\sqrt{x}}-3}{\sqrt[3]{x}-1}=\lim_{x\to 1}\frac{\sqrt{2x+\sqrt{x}}-3}{x-1}\times\frac{\sqrt{2x+\sqrt{x}}+3}{\sqrt{2x+\sqrt{x}}+3}\times\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$$

$$=\frac{2x+\sqrt{x}-9}{x-1}\times\frac{3}{4}\times\frac{(\sqrt{x}+1)(\sqrt{x}-4)}{(\sqrt{x}-1)(\sqrt{x}+1)}\times\frac{3}{3}=\frac{8}{2}\times\frac{3}{4}=\frac{12}{7}$$

(8)

$$\lim_{x\to\pi}\frac{1+\cos x}{\sin^2 x}=\lim_{x\to\pi}\frac{(1+\cos x)(1-\cos x+\cos^2 x)}{1-\cos^2 x}=\frac{(1+\cos x)(1-\cos x+\cos^2 x)}{(1-\cos x)(1+\cos x)}=\frac{3}{2}$$

(9)

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\lim_{x\to\frac{\pi}{4}}\frac{\frac{\cos x-\sin x}{\cos x}}{\sin x-\cos x}=\frac{-1}{\cos x}=\frac{-2}{\sqrt{2}}=-\sqrt{2}$$

(10)

$$\lim_{x\to\frac{\pi}{4}}\frac{\tan^2 x-1}{\cos 2x}=\frac{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}{\cos^2 x-\sin^2 x}=\frac{-1}{\cos^2 x}=-2$$