$$\lim_{x \to 1} \frac{4x^2 - 7x + 3}{5x^2 - 8x + 3} = \frac{(x-1)(4x-3)}{(x-1)(5x-3)} = \boxed{\frac{1}{2}}$$

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$$\lim_{x \to 0} \frac{|3x-1| - |3x+1|}{x}$$

$$0^+ \to = \frac{-(3x-1) - (3x+1)}{x} = \frac{-6x}{x} = \boxed{-6}$$

$$0^- \to = \frac{-(3x-1) - (3x+1)}{x} = \frac{-6x}{x} = \boxed{-6}$$

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$$\lim_{x \to 4} \frac{x-4}{\sqrt{x}-2} \Rightarrow \frac{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)}{\cancel{(\sqrt{x}-2)}} = \sqrt{x}+2 \Rightarrow = 2+2 = \boxed{4}$$

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$$\lim_{x \to 2} \frac{x-\sqrt{2x}}{2x^2-x-6} \Rightarrow = \frac{\sqrt{x}(\sqrt{x}-\sqrt{2})}{(2x+3)(x-2)} = \frac{(\sqrt{x})\cancel{(\sqrt{x}-\sqrt{2})}}{(2x+3)(\sqrt{x}+\sqrt{2})\cancel{(\sqrt{x}-\sqrt{2})}} = \frac{\sqrt{x}}{(2x+3)(\sqrt{x}+\sqrt{2})} = \frac{\sqrt{2}}{7(2\sqrt{2})} = \boxed{\frac{1}{14}}$$

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$$\lim_{x \to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} \Rightarrow = \frac{1-\sqrt{x}}{2-\sqrt{5-x}} \times \frac{2+\sqrt{5-x}}{2+\sqrt{5-x}} = \frac{(1-\sqrt{x})(2+\sqrt{5-x})}{4-5+x} = \frac{\underbrace{\cancel{(1-\sqrt{x})}}_{}(2+\sqrt{5-x})}{(\sqrt{x}+1)\underbrace{\cancel{(\sqrt{x}-1)}}_{-1}}$$

$$= \frac{2+\sqrt{5-x}}{-(\sqrt{x}+1)} = \frac{4}{-2} = \boxed{-2}$$

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$$\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3} \Rightarrow = \frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}\times\frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}\times\frac{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}$$

$$= \frac{3x+4-16}{5x+7-27} = \frac{3x-12}{5x-20} = \boxed{\frac{3}{5}}$$

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$$\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1} \Rightarrow = \frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}\times\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+1+\sqrt[3]{x}}\times\frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2} = \frac{(3x+\sqrt{x}-4)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}{(\sqrt{3x+\sqrt{x}}+2)(x-1)}\times\frac{(3x-4)-\sqrt{x}}{(3x-4)-\sqrt{x}}$$

$$\cancel{= \frac{(9x^2+16-24x-x)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}{(\sqrt{3x+\sqrt{x}}+2)(x-1)} = \frac{(9x^2-25x+16)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}{(\sqrt{3x+\sqrt{x}}+2)(x-1)}}$$

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$$\cancel{= \frac{(-7)(3)}{4}} = \frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}\times\frac{\sqrt[3]{x^2}+1+\sqrt[3]{x}}{\sqrt[3]{x^2}+1+\sqrt[3]{x}}\times\frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2} = \frac{(\sqrt[3]{x^2}+1+\sqrt[3]{x})(3x+\sqrt{x}-4)}{(\sqrt{x}-1)(\sqrt{x}+1)(\sqrt{3x+\sqrt{x}}+2)} = \frac{(3\sqrt{x}+4)(\sqrt[3]{x^2}+1+\sqrt[3]{x})}{(\sqrt{x}+1)(\sqrt{3x+\sqrt{x}}+2)} = \frac{21}{8}$$

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$$\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x} \Rightarrow = \frac{1+\cos^3 x}{\sin^2 x} \neq \frac{\cancel{(1+\cos x)}(\cos^2 x+1-\cos x)}{(1-\cos x)\cancel{(1+\cos x)}} = \frac{1+1+1}{2} = \boxed{\frac{3}{2}}$$

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$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x} \Rightarrow = \frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x} = \frac{\frac{\cos x-\sin x}{\cos x}}{\frac{\sin x-\cos x}{1}} = -\frac{1}{\cos x} = \boxed{-\sqrt{2}}$$

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$$\lim_{x\to\frac{3\pi}{4}}\frac{\tan^2 x-1}{\cos 2x} \Rightarrow = \frac{\frac{\sin^2 x}{\cos^2 x}-1}{\cos^2 x-\sin^2 x} = \frac{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}{\frac{\cos^2 x-\sin^2 x}{1}} = -\frac{1}{\cos^2 x} = \boxed{-2}$$

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