

طرح اولی در این مسئله

$$\lim_{x \rightarrow 1} \frac{x^2 - \sqrt{x+4}}{x^2 - 1} = \frac{(x-1)(x+1)}{(x-1)(x+1)} = \frac{x+1}{x+1} = 1$$

$$\lim_{x \rightarrow 0} \frac{|x-1| - |x+1|}{x} = \frac{-(x-1) - (x+1)}{x} = \frac{-2x}{x} = -2$$

$$\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} = \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = \sqrt{x}+2 = 4$$

$$\lim_{x \rightarrow 2} \frac{x - \sqrt{x}}{x^2 - x - 6} = x \frac{x + \sqrt{x}}{x + \sqrt{x}} = \frac{x^2 - 2x}{(x + \sqrt{x})(x^2 - x - 6)} = \frac{x(x-2)}{(x + \sqrt{x})(x-2)(x+3)} = \frac{1}{x+3} = \frac{1}{5}$$

$$\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{x - \sqrt{x}} = x \frac{1 + \sqrt{x}}{1 + \sqrt{x}} \frac{1 + \sqrt{0-x}}{1 + \sqrt{0-x}} = \frac{(1-x)(1 + \sqrt{0-x})}{(1-x)(1 + \sqrt{x})} = \frac{1}{2}$$

$$\lim_{x \rightarrow 4} \frac{\sqrt{x+4} - 4}{\sqrt{x} - 2} = \frac{\sqrt{x+4} + 4}{\sqrt{x} + 2} \times \frac{\sqrt{0x+4} + \sqrt{0x+4}}{\sqrt{0x+4} + \sqrt{0x+4}} = \frac{4x+16}{0x+8} = \frac{16}{8} = 2$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{x+\sqrt{x}} - 2}{\sqrt{x} - 1} = \frac{\sqrt{x+\sqrt{x}} + 2}{\sqrt{x} + 1} \times \frac{\sqrt{x^2 + \sqrt{x}} + 1}{\sqrt{x^2 + \sqrt{x}} + 1} = \frac{(x + \sqrt{x})(\sqrt{x^2 + \sqrt{x}} + 1)}{(\sqrt{x} + 1)(\sqrt{x^2 + \sqrt{x}} + 2)} = \frac{2}{3}$$

$$\lim_{x \rightarrow \pi} \frac{1 + \cos^2 x}{\sin^2 x} = \frac{(1 + \cos x)(1 + \cos x)}{(1 - \cos x)(1 + \cos x)} = \frac{1 + \cos x}{1 - \cos x}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \tan x}{\sin x - \cos x} = \frac{\cos x - \sin x}{\sin x - \cos x} = \frac{1}{-1} = -1$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan^2 x - 1}{\cos^2 x} = \frac{\sin^2 x - \cos^2 x}{-(\sin^2 x - \cos^2 x)} = \frac{-1}{-1} = 1$$