۱۷٫۲۵

نام و نام خانوادگی ...[illegible]... پاسخنامه تشریحی تکلیف شماره ...۳... کلاس ...یازدهم...

۱	$\lim\limits_{x\to 1}\dfrac{4x^2-7x+3}{5x^2-8x+3}=\dfrac{4\times1-7\times1+3}{5\times1-8\times1+3}=\dfrac{\text{صفر}}{\text{صفر}}\xrightarrow{\text{رفع ابهام}} 4x^2-7x+3\xrightarrow{a+b+c=0}\begin{cases}x=1\\ x=\frac{3}{4}\end{cases}$ $5x^2-8x+3\xrightarrow{a+b+c=0}\begin{cases}x=1\\ x=\frac{3}{5}\end{cases}$ $\lim\limits_{x\to 1}\dfrac{(x-1)(4x-3)}{(x-1)(5x-3)}\xrightarrow[(x-1)]{\text{حذف عامل مشترک}}\lim\limits_{x\to 1}\dfrac{4x-3}{5x-3}=\dfrac{1}{2}$ عامل مشترک $x-1$ در صورت و مخرج	۲
۲	$\lim\limits_{x\to 0}\dfrac{\lvert 3x-1\rvert-\lvert 3x+1\rvert}{x}=\dfrac{\lvert 3\times0-1\rvert-\lvert 3\times0+1\rvert}{0}=\dfrac{1-1}{0}=\dfrac{\text{صفر}}{\text{صفر}}$ $x=0 \Rightarrow x$ عامل مشترک صورت و مخرج $3x-1\xrightarrow{x\to0}3\times0-1=-1<0\xrightarrow{x\to0}\lvert 3x-1\rvert=-3x+1$ $3x+1\xrightarrow{x\to0}3\times0+1=1>0\xrightarrow{x\to0}\lvert 3x+1\rvert=3x+1$ یعنی x به صفر میل می‌کند $\lim\limits_{x\to 0}\dfrac{-3x+1-3x-1}{x}=\lim\limits_{x\to 0}\dfrac{-6x}{x}\xrightarrow{\text{حذف عامل مشترک}}\lim\limits_{x\to 0}\dfrac{-6x}{x}=-6$	۲
۳	$\lim\limits_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}=\dfrac{4-4}{\sqrt{4}-2}=\dfrac{\text{صفر}}{\text{صفر}}\xrightarrow{\text{رفع ابهام}}\lim\limits_{x\to 4}\dfrac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2}=\lim\limits_{x\to 4}\sqrt{x}+2=\sqrt{4}+2=4$	۲
۴	$\lim\limits_{x\to 2}\dfrac{x-\sqrt{2x}}{2x^2-x-6}=\dfrac{2-\sqrt{2\times2}}{2\times2^2-2-6}=\dfrac{\text{صفر}}{\text{صفر}}\xrightarrow{\text{رفع ابهام}}$ $x-2$ عامل مشترک صورت و مخرج $\begin{array}{r\|l} 2x^2-x-6 & x-2 \\ -2x^2+4x & 2x+3 \\ 3x-6 & \\ -3x+6 & \\ 0 & \end{array}$ $\lim\limits_{x\to 2}\dfrac{x-\sqrt{2x}}{(x-2)(2x+3)}=\lim\limits_{x\to 2}\dfrac{\sqrt{x}(\sqrt{x}-\sqrt{2})}{(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})(2x+3)}=\lim\dfrac{\sqrt{x}}{(\sqrt{x}+\sqrt{2})(2x+3)}=\dfrac{\sqrt{2}}{(\sqrt{2}+\sqrt{2})(\cancel{2\sqrt{2}+3})}$ ۲(۲)+۳ دقت! $\dfrac{\sqrt{2}}{2\sqrt{2}(2\sqrt{2}+3)}=\dfrac{1}{2(2\sqrt{2}+3)}\times\dfrac{3-2\sqrt{2}}{3-2\sqrt{2}}=\dfrac{3-2\sqrt{2}}{2(9-8)}=\cancel{\dfrac{3-2\sqrt{2}}{2}}$ $\boxed{\dfrac{1}{14}}$	۱٫۵
۵	$\lim\limits_{x\to 1}\dfrac{1-\sqrt{x}}{2-\sqrt{5-x}}=\dfrac{1-\sqrt{1}}{2-\sqrt{5}-1}=\dfrac{\text{صفر}}{1-\sqrt{5}}=0$ دقت!	۰

$$\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}=\frac{\sqrt{12+4}-4}{\sqrt[3]{20+7}-3}=\frac{\text{صفر}}{\text{صفر}}$$ رفع ابهام

$$\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}\times\underbrace{\frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}}_{a}\times\underbrace{\frac{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}}$$ ۶

$$=\lim_{x\to 4}\frac{\overset{-12}{3x+4-16}}{\underset{-20}{5x+7-27}}\times\frac{b}{a}=\lim_{x\to 4}\frac{3(x-4)}{5(x-4)}\times\frac{(\sqrt[3]{5x+7})^2+3\overset{b}{\sqrt[3]{5x+7}}+9}{\sqrt{3x+4}+4}=\frac{3}{5}\times\frac{(\sqrt[3]{27})^2+3\sqrt[3]{20+7}+9}{\sqrt{12+4}+4}=\frac{3}{5}\times\frac{9+9+9}{4+4}=\boxed{\frac{81}{40}}$$

(۲)

$$\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}=\frac{\sqrt{3+\sqrt{1}}-2}{\sqrt[3]{1}-1}=\frac{\text{صفر}}{\text{صفر}}\qquad\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}\times\frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2}\times\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$$

$$=\lim_{x\to 1}\frac{3x-4+\sqrt{x}}{x-1}\times\frac{3x-4-\sqrt{x}}{3x-4-\sqrt{x}}\times\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt{3x+\sqrt{x}}+2}=\lim_{x\to 1}\frac{9x^2\overbrace{-24x-x}^{-25x}+16}{x-1}\times\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{(\sqrt{3x+\sqrt{x}}+2)(3x-4-\sqrt{x})}$$ ۷

$$\lim_{x\to 1}\frac{(x-1)(9x-16)}{(x-1)}\times\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{(\sqrt{3x+\sqrt{x}}+2)(3x-4-\sqrt{x})}=9-16\times\frac{\sqrt[3]{1^2}+\sqrt[3]{1}+1}{(\underbrace{\sqrt{3+\sqrt{1}}+2}_{4})(\underbrace{3-4-\sqrt{1}}_{-2})}$$

$$=(\overset{-7}{\cancel{-5}})\times\frac{3}{4\times(-2)}=\boxed{\cancel{\frac{15}{8}}}\quad\frac{21}{8}$$

(۱.۷۵)

$$\lim_{x\to\pi}\frac{1+\cos^3x}{\sin^2x}=\frac{1+\cos^3\pi}{\sin^2\pi}=\frac{1-1}{0}=\frac{\text{صفر}}{\text{صفر}}\qquad\lim_{x\to\pi}\frac{(1+\cos x)(\cos^2x-\cos x+1)}{1-\cos^2x}$$

(۲)

$$=\lim_{x\to\pi}\frac{(1+\cos x)(\cos^2x-\cos x+1)}{(1+\cos x)(1-\cos x)}=\frac{\cos^2\pi-\cos\pi+1}{1-\cos\pi}=\frac{1+1+1}{1+1}=\boxed{\frac{3}{2}}$$ ۸

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\frac{1-1}{\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}}=\frac{\text{صفر}}{\text{صفر}}\qquad\lim_{x\to\frac{\pi}{4}}\frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x}=\lim_{x\to\frac{\pi}{4}}\frac{\overset{(-1)}{\frac{\cos x-\sin x}{\cos x}}}{\sin x-\cos x}$$

(۲)

$$=\lim_{x\to\frac{\pi}{4}}-\frac{1}{\cos x}=-\frac{1}{\frac{\sqrt{2}}{2}}=\boxed{-\sqrt{2}}$$ ۹

$$\lim_{x\to\frac{3\pi}{4}}\frac{\tan^2x-1}{\cos 2x}=\frac{\tan^2\frac{3\pi}{4}-1}{\cos\frac{3\pi}{2}}=\frac{1-1}{0}=\frac{\text{صفر}}{\text{صفر}}$$

(۲)

$$\lim_{x\to\frac{3\pi}{4}}\frac{\overset{-1}{\tan^2x-1}}{\frac{1-\tan^2x}{1+\tan^2x}}=(\tan^2x+1)(-1)=-(\tan^2\frac{3\pi}{4}+1)=\boxed{-2}$$ ۱۰

-۴

$$\xrightarrow{Hop} \lim_{x \to 2} \frac{1 - \frac{1}{\sqrt{2x}}}{4x - 1} = \frac{\frac{1}{2}}{7} = \frac{1}{14}$$

-۵

$$\xrightarrow{Hop} \lim_{x \to 1} \frac{\frac{-1}{2\sqrt{x}}}{-\frac{-1}{2\sqrt{5-x}}} = -2$$

-۷

$$\xrightarrow{Hop} \lim_{x \to 1} \frac{\frac{3 + \frac{1}{2\sqrt{x}}}{2\sqrt{3x + \sqrt{x}}}}{\frac{1}{3\sqrt[3]{x^2}}} = \frac{\frac{\frac{7}{2}}{4}}{\frac{1}{3}} = \frac{21}{8}$$