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$$\lim_{x\to 1}\frac{4x^2-7x+3}{5x^2-8x+3}$$

$$\longrightarrow = \frac{0}{0} \xrightarrow{\text{hop}} \frac{8x-7}{10x-8} = \frac{8-7}{10-8} = \frac{1}{2}$$

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$$\lim_{x\to 0}\frac{|[x]-1|-|[x+1]|}{x}$$

$$\begin{cases} 0^+ \longrightarrow \dfrac{[-1^+]-[1^+]}{0^+} = \dfrac{-2}{0^+} = -\infty \\ 0^- \longrightarrow \dfrac{[-1^-]-[1^-]}{0^-} = \dfrac{-2}{0^-} = +\infty \end{cases}$$

تابع در $x=0$ حد ندارد

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$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2} = \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = \frac{\sqrt{x}+2}{1} = \frac{\sqrt{4}+2}{1} = 4$$

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$$\lim_{x\to 2}\frac{x-\sqrt{2x}}{2x^2-x-6} \xrightarrow{\sqrt{2x}=4} \frac{x-2}{2x^2-x-6} = \frac{x-2}{(x-2)(2x+3)} = \frac{1}{2x+3} = \frac{1}{7}$$

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$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}} = \frac{1-x}{4-5+x} \times \frac{2+\sqrt{5-x}}{1+\sqrt{x}} = -\frac{2+\sqrt{5-x}}{1+\sqrt{x}} = -\frac{2+\sqrt{5-1}}{1+\sqrt{1}} = -\frac{4}{2} = -2$$

$$\lim_{x\to 2}\frac{\sqrt{3x+2}-2}{\sqrt[3]{3x+1}-3}=\frac{3x+2-16}{3x+7-27}\times\frac{\sqrt[3]{(3x+1)^2}+3\sqrt[3]{3x+1}+9}{\sqrt{3x+2}+4}=\frac{3x-12}{3x-20}\times\frac{27}{8}=\frac{3}{5}\times\frac{27}{8}=\frac{81}{40}=2.025$$ (۴)

$$\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x^2}}-2}{\sqrt[3]{x^2}-1}=\frac{3x+\sqrt{x^2}-4}{x-1}\times\frac{\sqrt[3]{x^4}+\sqrt[3]{x^2}+1}{\sqrt{3x+\sqrt{x^2}}+2}=\frac{(\sqrt{x}-1)(3\sqrt{x}+4)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}{(\sqrt[3]{x^2}-1)(\sqrt[3]{x^2}+1)}\times\frac{3}{4}=\frac{7}{2}\times\frac{3}{4}=\frac{21}{8}=2.625$$ (۷)

$$\lim_{x\to\pi}\frac{1+\cos^2 x}{\sin^2 x}=\frac{(1+\cos)(1-\cos+\cos^2)}{1-\cos^2}=\frac{(1+\cos)(1-\cos+\cos^2)}{(1+\cos)(1-\cos)}=\frac{1-\cos x+\cos^2 x}{1-\cos x}=\frac{1-(-1)+1}{1-(-1)}=\frac{3}{2}$$ (۱۱)

$\cos\pi=-1$

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\frac{\frac{\cos}{\cos}-\frac{\sin}{\cos}}{\sin-\cos}=\frac{\cos-\sin}{(\sin-\cos)\cos}=\frac{-1}{\cos x}=\frac{-1}{\frac{\sqrt{2}}{2}}=-\sqrt{2}$$ (۱۹)

$\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$$\lim_{x\to\frac{3\pi}{4}}\frac{\tan^2 x-1}{\cos 2x}=\frac{\frac{\sin^2}{\cos^2}-\frac{\cos^2}{\cos^2}}{\cos^2-\sin^2}=\frac{\sin^2-\cos^2}{\cos^2(\cos^2-\sin^2)}=\frac{-1}{\cos^2 x}=\frac{-1}{\frac{1}{2}}=-2$$ (۱۱۰)

$\cos\frac{3\pi}{4}=\frac{-\sqrt{2}}{2}$

$\cos^2\frac{3\pi}{4}=\frac{1}{2}$