

A, 5

نام و نام خانوادگی: ... پاسخنامه تشریحی تکلیف شماره ۴ کلاس: ...

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{x^2 - 2x + 1 - x^2}{x(x-1)}} = \sqrt{\frac{-2x+1}{x(x-1)}} \rightarrow \frac{1}{x}$

$D_f = (-\infty, 0) \cup (\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}} = \frac{\frac{x - 2(x+1)}{x(x+1)}}{\frac{3(x+2) + (x-1)}{(x-1)(x+2)}} = \frac{\frac{-x-2}{x(x+1)}}{\frac{4x+5}{(x-1)(x+2)}} = \frac{-(x+2)(x-1)}{x(x+1)(4x+5)}$

$D_f = (-\frac{5}{4}, -1) \cup (0, 1)$

تأخیر این ریشه‌ها را از R کم کن

الف) $f(x) = \sqrt{((\frac{1}{x})^2 - 9)(32 - 4x)} = \sqrt{(\frac{1}{x^2} - 9)(32 - 4x)}$

$D_f = (-\infty, -2] \cup [\frac{8}{3}, +\infty)$

چون فقط خروجی‌ها نباید منفی شوند!

ب) $\sqrt{x-1} + \sqrt{y+1} = 3$

حد اول $= 0 \rightarrow x=1$
حد آخر $= 3 \rightarrow x=12$

$D_y = [1, 12]$

x منفی + y منفی $= 3$

$f(x) = \frac{\log_4(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

① $\rightarrow x^2 - 1 \geq 0 \rightarrow (x-1)(x+1) \geq 0 \rightarrow x \in (-\infty, -1] \cup [1, +\infty)$

② $\sqrt{x^2 - 1} + 1 \neq 0 \rightarrow$ موارد برقرار

③ $\rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$

$x \in (-\infty, -1) \cup (2, +\infty)$

① ② ③ $\rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$D_f = [-2, b]$ $f(x) = \sqrt{4ax - x^2}$

یعنی دو سر دامنه ریشه اند پس $x = -2$ if $f(x) = 0$

$\rightarrow f(-2) = \sqrt{3 - 2a - 4} = 0 \rightarrow a = -\frac{1}{2}$

$\rightarrow$ ضرب ریشه $= \frac{c}{a} = \frac{3}{-\frac{1}{2}} = -6 = -2 \times b \rightarrow b = 3$

$\rightarrow a+b = -\frac{1}{2} + 3 = \frac{5}{2}$

$f(x) = \begin{cases} 3x-2 & ; x \geq 1 \\ 2x+3 & ; x < 1 \end{cases} \rightarrow g(x) = \begin{cases} \sqrt{3x-2} & ; x \geq 1 \\ \sqrt{x+3} & ; x < 1 \end{cases}$

① $\rightarrow 3x-2 \geq 0 \rightarrow x \geq \frac{2}{3}, x \geq 1 \rightarrow \mathbb{R}$

② $\rightarrow x+3 \geq 0 \rightarrow x \geq -3, x < 1 \rightarrow x \in [-3, 1)$

① $\cup$ ② $\rightarrow D_g = [-3, 1) \cup [1, +\infty) = [-3, +\infty)$

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ (2a+2x) & ; x \leq 1 \end{cases}$$

$$2f(\Delta) = f(-2) + a$$

(2)

$$\begin{aligned} \rightarrow 2f(\Delta) &= 2((a+1)(2)) = 4a+4 \\ \rightarrow f(-2) &= 2a-2 \end{aligned} \quad \left. \begin{aligned} 10a \\ 4a+4 \\ 2a-2 \end{aligned} \right\} \begin{aligned} 10a \\ 4a+4 \\ 2a-2 \end{aligned} = \frac{-11}{\Delta}$$

$$\rightarrow 10a = -11 \rightarrow a = -1.1 = -\frac{11}{10} \checkmark$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(2-\sqrt{2}) + f(2+\sqrt{2})$$

(2)

$$\sqrt{2+\sqrt{2}} + \frac{1}{\sqrt{2+\sqrt{2}}} + 2 + \sqrt{2-\sqrt{2}} + \frac{1}{\sqrt{2-\sqrt{2}}} + 2 = \frac{2+\sqrt{2}+1}{\sqrt{2+\sqrt{2}}} + \frac{2-\sqrt{2}+1}{\sqrt{2-\sqrt{2}}} + 4$$

می دانیم $2+\sqrt{2}$ و $2-\sqrt{2}$ معکوس یکدیگرند زیرا $(2+\sqrt{2})(2-\sqrt{2}) = 4-2 = 2$
if $a=2+\sqrt{2}$ یک می شود.

$$\rightarrow \frac{a+1}{\sqrt{a}} + \frac{\frac{1}{a}+1}{\sqrt{\frac{1}{a}}} + 4 = \frac{\sqrt{a}+\sqrt{\frac{1}{a}}}{1} + 4$$

$$\rightarrow 2\sqrt{2+\sqrt{2}} + 2\sqrt{2-\sqrt{2}} + 4 = \frac{\sqrt{2+\sqrt{2}} + \sqrt{2-\sqrt{2}}}{1} + 4 \quad \text{if } \sqrt{2+\sqrt{2}} + \sqrt{2-\sqrt{2}} = 2$$

$$\begin{aligned} 2f(x) - 3f(-x) &= 4x^2 - x \xrightarrow{\times 2} 4f(x) - 6f(-x) = 8x^2 - 2x \\ 2f(-x) - 3f(x) &= 4x^2 + x \xrightarrow{\times 2} 4f(-x) - 6f(x) = 8x^2 + 2x \end{aligned}$$

(2)

$$\rightarrow -2f(x) = 20x^2 + x$$

$$\rightarrow f(x) = -10x^2 - \frac{x}{2} \checkmark$$

$$(n+2)f(n) - 3nf(n+2) = 4n^2 - mn + 2m-1$$

$$2f(0) = 2m-1 \rightarrow f(0) = \frac{2m-1}{2}$$

$$4f(0) = 14 + 2m + 2m-1 = 14 + 4m-1 \rightarrow f(0) = \frac{14 + 4m-1}{4}$$

$$\rightarrow \frac{2m-1}{2} = \frac{14 + 4m-1}{4} \rightarrow 4m-2 = 14 + 4m \rightarrow 4m = 16$$

$$\rightarrow m = 4 \checkmark \rightarrow f(0) = \frac{2m-1}{2} = \frac{8-1}{2} = \frac{7}{2} = 3.5 \checkmark$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x}$$

می دانیم تابع $f(x)$ خطی است

$$\rightarrow f(x) = ax + b \rightarrow f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b$$

$$\rightarrow \frac{ax^2 + bx + a}{x} = \frac{3x^2 - 12x + 3}{x} \rightarrow a=3 \checkmark \quad b=-9 \checkmark$$

$$\rightarrow f(x) = 3x - 9 \rightarrow f(-1) = -3 - 9 = -12 \checkmark$$

⑤ $n^2 = 2 + \sqrt{2} + 2 - \sqrt{2} + 2\sqrt{2-2} = 4 \rightarrow n = \sqrt{4}$

$$\rightarrow f(2+\sqrt{2}) + f(2-\sqrt{2}) = 2(\sqrt{4}) + 4 = 12 \checkmark$$