

1- (۲) $f(x) = \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}}$ $\rightarrow$ مرتب می کنیم

$\rightarrow \frac{-x+1}{x(x-1)} \rightarrow \frac{0}{x} + \frac{1}{x-1} + \frac{1}{x} = Df \in (-\infty, 0) \cup [\frac{1}{4}, \infty)$ ✓

ب) $f(x) = \frac{1}{x+1} - \frac{2}{x} \rightarrow x \neq -1, 0$
 $\frac{3}{x-1} + \frac{1}{2x} \rightarrow x \neq 1, -2$

$\rightarrow$ $\frac{3}{x-1} + \frac{1}{2x} \neq 0 \rightarrow \frac{3x+4+x-1}{(x-1)(2x)} = \frac{4x+3}{(x-1)(2x)} \neq 0$
 $\frac{-2 - \frac{3}{x}}{-1 + \frac{1}{x}} \rightarrow Df \in \mathbb{R} - \left\{ -2, -\frac{4}{3}, 1, 0 \right\}$ ✓

۲- $f(x) = \sqrt{\frac{-x^2}{(x^2-1)(x^2-4)}}$
 $Df \in (-\infty, -2] \cup [\frac{4}{3}, \infty)$ ✓

ب) $\sqrt{x-1} + \sqrt{y+1} \leq 3 \rightarrow x \in (-1, 0) \cup (0, 1)$

$3 - \sqrt{x-1} \geq 0 \rightarrow x \leq 10$

$Df \in [1, 10]$

$$f(x) = \frac{\log y}{\sqrt{x^2 - 1} + 1}$$

(۲)

$$\rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$$

$$x^2 - 1 > 0 \rightarrow (x-1)(x+1) > 0$$

$$Df \in (-\infty, -1) \cup (2, \infty) \checkmark$$

$$f(x) = \sqrt{-x^2 + ax + 3}$$

(۲)

چون تابع حقیقی باشد پس حاصله در زیر رادیکال باید مثبت باشد

$$\alpha \beta \leq \frac{c}{a} \leq -3 \rightarrow b \leq \frac{3}{4} \checkmark$$

$$\alpha + \beta \leq -\frac{a}{-1} \leq \frac{a}{1} \leq \frac{3}{4} + 1 - 2 \leq \frac{-1}{4} \rightarrow a \leq -\frac{1}{4} \checkmark$$

$$a + b \leq \frac{1}{4} + \frac{3}{4} \leq \frac{4}{4} \leq 1 \checkmark$$

$$f(x) = \frac{x(1-x)}{x+1} \quad (2)$$

$$f(x) = \frac{x(1-x)}{x+1} \Rightarrow x(1-x) = (x+1)f(x)$$

$$x(1-x) = x + xf(x)$$

$$x - x^2 = x + xf(x)$$

$$-x^2 = xf(x)$$

$$f(x) = -x \quad \text{for } x \in (-1, 1) \cup [1, +\infty)$$

$$f(1) = 1 + 1 = 2$$

$$f(x) = -x + 1 \quad (1, \infty)$$

$$\Rightarrow f(1) = f(-1) + 1 \Rightarrow 2 = 1 + 1$$

$$\Rightarrow 1 + 1 = 2 \Rightarrow 1 + 1 = 2 \Rightarrow 1 + 1 = 2$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 1 \Rightarrow \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 + 1$$

$$= \sqrt{x + \frac{1}{x} + 1} + 1 \Rightarrow f(x + \frac{1}{x}) = \sqrt{x + \frac{1}{x} + 1} + 1$$

$$f(x + \frac{1}{x}) = \sqrt{x + \frac{1}{x} + 1} + 1$$

$$f(x + \frac{1}{x}) + f(x + \frac{1}{x}) = 2\sqrt{x + \frac{1}{x} + 1} + 2$$

$$(f(x) - f(-x)) \leq (x^2 - 2) \alpha^2$$

$$(f(-x) - f(x)) \leq (x^2 + 2) \alpha^2$$

(2)

$$\Rightarrow f(x) - f(-x) \leq x^2 - 2\alpha^2$$

$$\Rightarrow f(-x) - f(x) \leq x^2 + 2\alpha^2$$

$$- \Delta f(x) \leq x^2 + 2\alpha^2$$

$$\Rightarrow f(x) \leq \frac{x^2 + 2\alpha^2}{-1} \checkmark$$

$$\forall x \leq 0 \Rightarrow f(x) \leq x^m - 1$$

(2)

-9

$$\forall x \leq -1 \Rightarrow f(x) \leq 1 + x^m + x^{m-1}$$

$$9m - 1$$

$$\leq 9m + 1$$

$$(x^m - 1) \alpha^2 \leq 9m + 1 \Rightarrow x^m \leq 1 \Rightarrow m \leq \frac{9}{1} \checkmark$$

$$f(0) \leq 1 \left(\frac{9}{1} \right) - 1 \leq \frac{9}{1} \Rightarrow f(0) \leq \frac{9}{1} \checkmark$$

$$\forall x \leq -1 \Rightarrow f(-1) \leq \frac{1 + 1 + 1}{-1} = -3$$

(2)

-10

$$f(-1) \leq -3 \checkmark$$