

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{1-x^2}{x(x-1)} \geq 0$
 $x \neq 0, 1 \Rightarrow \frac{0}{+} - \frac{+}{+} \rightarrow D_f = (-\infty, 0) \cup [\frac{1}{x}, 1)$

ب) $f(x) = \frac{1}{x+1} - \frac{x}{x}$ $\rightarrow x \neq -1, 0$ و $-2 \Rightarrow \frac{x}{x-1} + \frac{1}{x+1} = \frac{x^2 - \frac{1}{x}}{(x-1)(x+1)}$ $D_f = \mathbb{R} \setminus \{-1, -\frac{1}{x}, 0\}$

الف) $f(x) = \sqrt{(\frac{1}{x})^2 - 9} (3^2 - 4x) \rightarrow (\frac{1}{x})^2 - 9 \geq 0 \rightarrow \begin{cases} (\frac{1}{x})^2 - 9 = 0 \rightarrow x = -\frac{1}{3} \\ 3^2 - 4x = 0 \rightarrow x = \frac{9}{4} \end{cases}$
 $\rightarrow \frac{-}{+} - \frac{+}{+} \Rightarrow D_f = (-\infty, -\frac{1}{3}] \cup [\frac{9}{4}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \begin{cases} x-1 \geq 0 \Rightarrow x \geq 1 \\ y+1 \geq 0 \Rightarrow y \geq -1 \end{cases} \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \xrightarrow{y+1 \geq 0} 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x \leq 10$

① ∩ ② $\rightarrow D_f = [1, 10]$

$f(x) = \frac{\log_e(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow \frac{+}{+} - \frac{+}{+} \rightarrow$ ①
 $\sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1 \Rightarrow x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1, x \leq -1$ ②
 $\textcircled{1} \cap \textcircled{2} \rightarrow D_f = (-\infty, -1) \cup (1, +\infty)$

$f(x) = \sqrt{3+ax} - x^2$ $D_f = [-1, b]$
 $\Rightarrow 3+ax - x^2 \geq 0 \rightarrow -x^2 + ax + 3 = -(x+1)(x-b) \Rightarrow \frac{+}{-} - \frac{+}{-} = -x^2 + (b-1)x + 3$
 $b = 3 \rightarrow b = \frac{3}{1}$
 $a = b - 1 \rightarrow a = \frac{3}{1} - 1 = 2$
 $a + b = -\frac{1}{1} + \frac{3}{1} = 2$

$f(x) = \begin{cases} 3x-1; & x \geq 1 \\ 1x+3; & x < 1 \end{cases}$ $g(x) = \sqrt{f(x)} - x$

$x \geq 1 \rightarrow f(x) - x = 3x-1-x = 2x-1 = 2(x-\frac{1}{2}) \rightarrow 2(x-\frac{1}{2}) \geq 0 \Rightarrow [1, +\infty)$ ①

$x < 1 \rightarrow f(x) - x = 1x+3-x = 3 \rightarrow 3 \geq 0 \Rightarrow x \in (-\infty, 1) \rightarrow [-3, 1)$ ②

① ∪ ② $D_g = [-3, +\infty)$

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