

الف) $f(m) = \sqrt{\frac{m-1}{m} - \frac{m}{m-1}}$ $\xrightarrow{D_f}$ $\begin{cases} \text{ریشه یکنوا} \neq 0 \\ \text{و} > 0 \end{cases} \rightarrow \begin{cases} \text{I} \begin{cases} m+0 \\ m \neq 1 \end{cases} \\ \text{II} \frac{m-1}{m} - \frac{m}{m-1} > 0 \rightarrow \frac{m^2+1-2m-m^2}{m(m-1)} > 0 \end{cases}$ سوال ۱

$\Rightarrow \frac{-2m+1}{m(m-1)} > 0 \xrightarrow{\text{ریشه ها}} \begin{cases} m = \frac{1}{2} \\ m = 0 \\ m = 1 \end{cases} \rightarrow \begin{array}{c} 0 \quad \frac{1}{2} \quad 1 \\ + \quad - \quad + \quad - \end{array} \xrightarrow{\text{اشتراک}} \begin{array}{c} 0 \quad 1 \\ + \quad - \end{array}$

$\Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(m) = \frac{1}{m+1} - \frac{1}{m} \xrightarrow{\text{تکلیف سری}} \frac{1}{m-1} + \frac{1}{m+2} \xrightarrow{\text{ریشه یکنوا} \neq 0} \begin{cases} m+1 \neq 0 \rightarrow m \neq -1 \\ m \neq 0 \rightarrow m \neq 0 \\ m-1 \neq 0 \rightarrow m \neq 1 \\ m+2 \neq 0 \rightarrow m \neq -2 \end{cases}$

$\frac{1}{m-1} + \frac{1}{m+2} \neq 0 \rightarrow \frac{m+2+m-1}{(m-1)(m+2)} = \frac{2m+1}{(m-1)(m+2)} \neq 0 \rightarrow \frac{2m+1}{(m-1)(m+2)} \neq 0 \rightarrow m \neq -\frac{1}{2}$

$\Rightarrow D_f = \mathbb{R} - \{-2, -\frac{1}{2}, -1, 0, +1\}$

الف) $f(m) = \sqrt{((\frac{1}{2})^m - 9)(32 - 4^m)}$ $\xrightarrow{\text{ریشه یکنوا} > 0} \begin{cases} (\frac{1}{2})^m - 9 > 0 \\ (32 - 4^m) > 0 \end{cases}$ سوال ۲

$\Rightarrow \begin{array}{c} -2 \quad +\frac{5}{2} \\ + \quad - \quad + \end{array} \xrightarrow{\text{تعیین علامت}} \begin{array}{c} m = -2 \quad \leftarrow \quad \rightarrow \quad m = \frac{5}{2} \end{array} \Rightarrow D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب) $\sqrt{m-1} + \sqrt{y+1} = 3 \xrightarrow{\text{شرایط}} \begin{cases} m-1 > 0 \rightarrow m > 1 \\ y+1 > 0 \rightarrow y > -1 \end{cases} \xrightarrow{\text{I}} \sqrt{y+1} = 3 - \sqrt{m-1} \rightarrow 3 > \sqrt{m-1}$

$\xrightarrow{\text{مربع کردن طرفین}} 9 > m-1 \xrightarrow{\text{توان ۲}} 10 > m \xrightarrow{\text{II}} m < 10 \xrightarrow{\text{اشتراک I و II}} \begin{cases} m > 1 \\ m < 10 \end{cases}$

$D_f = [1, 10]$

الف) $f(m) = \frac{\log(m^2 - m - 2)}{\sqrt{m^2 - 1} + 1} \xrightarrow{\text{تکلیف سری}} \begin{cases} \text{ریشه یکنوا} > 0 \\ \text{و} > 0 \end{cases} \rightarrow \begin{cases} m^2 - 1 = (m-1)(m+1) > 0 \\ m^2 - m - 2 > 0 \end{cases}$ سوال ۳

$m^2 - m - 2 = (m-2)(m+1) > 0 \rightarrow \begin{array}{c} -1 \quad +2 \\ + \quad - \quad + \end{array} \xrightarrow{\text{اشتراک}} \begin{array}{c} -1 \quad +1 \quad +2 \\ + \quad - \quad + \end{array} \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

سوال ۴

$$f(m) = \sqrt{3 + am - m^2} \rightarrow \phi = [-r, b]$$

$$a + b = ?$$

ریشه ها

$$-m^2 - \frac{1}{4}m + 3 = 0 \quad \begin{array}{c|c|c} - & + & - \\ \hline & & \end{array}$$

①

$$m = -r \quad -(-r)^2 + a(-r) + 3 = -r^2 - ra + 3 = -ra - 1 = 0 \rightarrow -ra = 1 \rightarrow a = -\frac{1}{r}$$

$$\Rightarrow -m^2 - \frac{1}{4}m + 3 \xrightarrow{\text{لایحه}} x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\frac{1}{4} \pm \sqrt{\frac{1}{16} + 12}}{-2} = \frac{\frac{1}{4} \pm \frac{\sqrt{193}}{4}}{-2} \in \left(-\frac{2}{3}, -\frac{2}{3}\right) \rightarrow \begin{cases} a = -\frac{1}{r} \\ b = -\frac{3}{r} \end{cases}$$

$$(a+b) = -\frac{4}{r} = -2$$

سوال ۵

$$f(m) = \begin{cases} (a+1)(m+2) & ; m \geq 1 \\ 2m+3 & ; m < 1 \end{cases} \quad g(m) = \sqrt{f(m)-m}$$

$$f(m)-m = \begin{cases} 3a-2-m & ; m \geq 1 \\ 2m+3-m & ; m < 1 \end{cases} = \begin{cases} 3m-2 & ; m \geq 1 \\ m+3 & ; m < 1 \end{cases}$$

خطی اینکه متوجه شویم این تابع در چه بازه ای نامنفذ است شکلش را رسم می کنیم که طبق شکل در بازه $[-3, +\infty)$ نامنفذ است

جواب $\rightarrow [-3, +\infty)$

سوال ۶

$$f(m) = \begin{cases} (a+1)(m+2) & ; m \geq 1 \\ 3a+2m & ; m < 1 \end{cases} \quad r f(a) = f(-r) + a$$

$$a = ? \rightarrow r(a+1)(r) = 3a + 2(-r) + a$$

$$1ra + 1r = 4a - 2 \Rightarrow 10a = -18 \rightarrow a = -1.8$$

سوال ۷

$$f(m) = \sqrt{m} + \frac{1}{\sqrt{m}} + 2$$

$$f(r-\sqrt{3}) + f(r+\sqrt{3}) = ?$$

$$\begin{cases} f(r-\sqrt{3}) = \sqrt{r-\sqrt{3}} + \frac{1}{\sqrt{r-\sqrt{3}}} + 2 \\ f(r+\sqrt{3}) = \sqrt{r+\sqrt{3}} + \frac{1}{\sqrt{r+\sqrt{3}}} + 2 \end{cases} \xrightarrow{\text{مکس}} \begin{cases} a \times \frac{1}{a} = 1 \rightarrow \text{مکس} \\ (r-\sqrt{3})(r+\sqrt{3}) = 1 \rightarrow \text{مکس} \end{cases}$$

$$\Rightarrow \begin{cases} f(r-\sqrt{3}) = \sqrt{r-\sqrt{3}} + \sqrt{r+\sqrt{3}} + 2 \\ f(r+\sqrt{3}) = \sqrt{r+\sqrt{3}} + \sqrt{r-\sqrt{3}} + 2 \end{cases} \Rightarrow f(r-\sqrt{3}) + f(r+\sqrt{3}) = 2(\sqrt{r-\sqrt{3}} + \sqrt{r+\sqrt{3}}) + 4$$

$$A^2 = r - \sqrt{3} + r + \sqrt{3} + 2\sqrt{(r-\sqrt{3})(r+\sqrt{3})} = 4 + 2 = 4 \Rightarrow A = \sqrt{4}$$

$2A + 4 = 2\sqrt{4} + 4$

سوال ۸

$$r f(m) - 3 f(-m) = 4m^2 - m$$

$$f(m) = ? \begin{cases} \xrightarrow{x} r f(m) - 3 f(-m) = 4m^2 - m \\ \xrightarrow{-m} r f(-m) - 3 f(m) = 4m^2 + m \end{cases} \rightarrow \begin{cases} \times 2 \rightarrow 4 f(m) - 4 f(-m) = 8m^2 - 2m \\ \times 3 \rightarrow 3 r f(-m) - 9 f(m) = 12m^2 + 3m \end{cases}$$

$$-5 f(m) = 20m^2 + m$$

$$f(m) = \frac{20m^2 + m}{-5} \rightarrow -4m^2 - \frac{1}{5}m = f(m)$$

$$(n+2)f(n) - 3nf(n+2) = 5n^2 - mn + 3m-1$$

$$f(0) = ?$$

سوال 9 ←

$$\left\{ \begin{array}{l} n=0 \rightarrow 2f(0) - 3(0)f(2) = 5(0)^2 - m(0) + 3m-1 \Rightarrow 2f(0) = 3m-1 \\ \Rightarrow f(0) = \frac{3m-1}{2} \end{array} \right.$$

$$\left\{ \begin{array}{l} m=-1 \rightarrow (-2+2)f(-1) - 3(-1)f(0) = 5(-1)^2 - m(-1) + 3m-1 \\ \quad + 4f(0) = 14 + 2m + 3m-1 = 15 + 5m \rightarrow f(0) = \frac{15+5m}{4} = \frac{3m-1}{2} = 9m-3 = 5m+12 \end{array} \right.$$

$$\Rightarrow 5m = 12 \rightarrow m = \frac{12}{5} = \frac{4}{1} = 4, 25 \Rightarrow f(0) = \frac{3m-1}{2} = \frac{3(4,25)-1}{2} = \frac{12,75-1}{2} = 4,25$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{5x^2 - 12x + 3}{x} \xrightarrow{-1 \text{ ليبد}} 2f(-1) + f(-1) = \frac{5(-1)^2 - 12(-1) + 3}{-1} = \frac{5+12+3}{-1} = -20 \leftarrow \text{سوال 10}$$

$$\Rightarrow 2f(-1) = -20 \rightarrow f(-1) = -10$$