

$$\begin{aligned} \text{الف) } x \neq 0, x-1 \neq 0, \quad \frac{x-1}{x} - \frac{x}{x-1} \geq 0 &\Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{-4x+1}{x(x-1)} \geq 0 \\ \frac{x}{x+1} \geq \frac{1}{x-1} &\Rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{4}, 1\right) \end{aligned}$$

$$\begin{aligned} \text{b.) } & \boxed{x \neq -1}, \boxed{x \neq 0}, \boxed{x-1 \neq 0}, \boxed{x+1 \neq 0}, \boxed{x+y \neq 0}, \boxed{x \neq -y} \quad \left| \quad \frac{4}{x-1} + \frac{1}{x+y} \neq 0 \Rightarrow \frac{4(x+y) + x-1}{(x-1)(x+y)} \neq 0 \right. \\ & \Rightarrow \frac{5x+3}{(x-1)(x+y)} \neq 0 \Rightarrow \boxed{x = \frac{-3}{5}} \end{aligned}$$

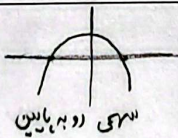
$$\text{الف) } \left(\left(\frac{1}{x}\right)^x - q\right)(y - \epsilon^x) \geq 0 \Rightarrow (y^x - y^y)(y^0 - y^{yx}) \geq 0 \quad y > 1 \Rightarrow y^x \geq y^y \Leftrightarrow x \geq y$$

$$\rightarrow \frac{x}{+} \mid \frac{-y}{+} \frac{0}{-} \mid \frac{0}{+} \rightarrow D_f = (-\infty, -y] \cup \left[\frac{0}{y}, +\infty\right)$$

$$\left. \begin{aligned} \sqrt{y+1} \geq 0 &\Rightarrow \sqrt{x-1} \leq 12 \Rightarrow x-1 \leq 144 \Rightarrow x \leq 145 \\ x-1 \geq 0 &\Rightarrow x \geq 1 \end{aligned} \right\} \Rightarrow 1 \leq x \leq 145 \Rightarrow D_f = [1, 145]$$

$$\begin{aligned}
 & x^2 - x - y > 0 \\
 & (x-y)(x+1) \geq 0 \rightarrow \left. \begin{array}{l} \begin{array}{c} x \mid \begin{array}{ccc} -1 & & 1 \\ + & - & - \\ \hline & + & - \end{array} \end{array} \quad \text{(I)} \\
 & \quad \quad \quad x \in (-\infty, -1) \cup (1, +\infty) \\
 & x^2 - 1 \geq 0 \\
 & (x-1)(x+1) \geq 0 \rightarrow \left. \begin{array}{l} \begin{array}{c} x \mid \begin{array}{ccc} -1 & & 1 \\ + & - & - \\ \hline & + & - \end{array} \end{array} \quad \text{(II)} \\
 & \quad \quad \quad x \in (-\infty, -1] \cup [1, +\infty) \\
 & \sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1 \rightarrow x \in \mathbb{R} \quad \text{(III)} \\
 & \sqrt{x} \geq 0
 \end{aligned}
 \right\}
 \begin{aligned}
 & \text{I} \cap \text{II} \cap \text{III} = (-\infty, -1) \cup (1, +\infty) \\
 & \boxed{D_f = (-\infty, -1) \cup (1, +\infty)}
 \end{aligned}$$

$$f(x) = -x^2 + ax + b \geq 0$$



$$\Rightarrow D_f = [x_1, x_2], f(x_1) = f(x_2) = 0, x_1 < x_2$$

$$\rightarrow f(-1) = 0 \Rightarrow -(-1)^4 + \alpha(-1) + 1 = 0 \Rightarrow -1 - \alpha + 1 = 0 \Rightarrow \alpha = -1 \Rightarrow \boxed{p = \frac{-1}{x}}$$

$$f(b) = 0 \Rightarrow p = b \cdot (-1) = \frac{c}{a} = \frac{p}{-1} = -p \Rightarrow \boxed{b = \frac{p}{-1}}$$

$$a+b = \frac{-1}{y} + \frac{y}{y} = \boxed{1}$$

$$f(x) \geq x$$

$$\textcircled{1} \quad x \geq 1 \Rightarrow f(x) = x^x - x \geq x \Rightarrow x \geq 1 \Rightarrow \begin{cases} x \geq 1 \\ x \geq 1 \end{cases} \Rightarrow \boxed{x \geq 1} \quad \text{I}$$

$$\textcircled{1} x \leq 1 \Rightarrow f(x) = 1x + 4 \geq x \Rightarrow \left. \begin{array}{l} x \geq -4 \\ \textcircled{1} x \leq 1 \end{array} \right\} \boxed{-4 \leq x \leq 1} \quad \text{II}$$

$$D_f =]0, 1[= [-3, +\infty)$$

$$\omega > 1 \Rightarrow f(\omega) = (\alpha + 1)(0 + 1) = \alpha + 1 \Rightarrow \boxed{4f(\omega) = 16\alpha + 16}$$

$$-1 < 1 \Rightarrow f(-1) = 3\alpha + 4(-1) = \boxed{3\alpha - 4}$$

$$\Rightarrow 4f(\omega) = f(-1) + \alpha \Rightarrow 16\alpha + 16 = 3\alpha - 4 + \alpha = 4\alpha - 4$$

$$\Rightarrow 10\alpha = -20 \Rightarrow \boxed{\alpha = -2}$$

$$\frac{1}{1+\sqrt{x}} = \frac{1(1-\sqrt{x})}{(1+\sqrt{x})(1-\sqrt{x})} = 1-\sqrt{x} \quad \left\{ \begin{array}{l} f(1+\sqrt{x}) + f(1-\sqrt{x}) = 1(\sqrt{1+\sqrt{x}} + \sqrt{1-\sqrt{x}} + 1) = \\ f + 1(\sqrt{1+\sqrt{x}} + \sqrt{1-\sqrt{x}}) = f + 1(\sqrt{(1+\sqrt{x})(1-\sqrt{x})}) = \\ f + 1(\sqrt{1-x+\sqrt{x}-x}) = f + 1(\sqrt{1-x-\sqrt{x}}) = \boxed{f + 1\sqrt{1-x}} \end{array} \right.$$

$$4f(x) - 4f(-x) = 16x^2 - 16 : P(x)$$

$$P(-x) \Rightarrow 4f(-x) - 4f(x) = 16(-x)^2 - 16(-x) = 16x^2 + 16x$$

$$P(x) \Rightarrow P(-x) \Rightarrow \left\{ \begin{array}{l} 4f(x) - 4f(-x) = 16x^2 - 16x \\ 4f(-x) - 4f(x) = 16x^2 + 16x \end{array} \right\} \Rightarrow -8f(x) = 16x^2 + 16x \Rightarrow \boxed{f(x) = -2x^2 - 2x}$$

$$\left. \begin{array}{l} P(0) \Rightarrow 4f(0) = 16m - 1 \Rightarrow 4f(0) = 16m - 1 \\ P(-1) \Rightarrow 4f(0) = 16(-1)^2 - 16(-1) + 16m - 1 = 16 + 16m \end{array} \right\} \begin{array}{l} 16m - 1 = 16 + 16m \\ 16m = 17 \\ \boxed{m = 17/16} \end{array}$$

$$\Rightarrow 4f(0) = 16m - 1 = 17 \times 16 - 1 = 260$$

$$\Rightarrow \boxed{f(0) = 65}$$

$$\left. \begin{array}{l} f(x) = ax + b \\ f(1/x) = \frac{a}{x} + b \end{array} \right\} \Rightarrow f(x) + f(1/x) = a(x + \frac{1}{x}) + 2b = \frac{ax^2 + 1bx + a}{x} = \frac{3x^2 - 12x + 3}{x}$$

$$\Rightarrow \boxed{a = 3} \quad 1b = -12 \Rightarrow \boxed{b = -12}$$

$$f(x) = ax + b = 3x - 12$$

$$f(-1) = 3(-1) - 12 = \boxed{-15}$$