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باسمه تعالی

نام و نام خانوادگی: علی ربانی

پاسخنامه تکلیف شماره ۴ — کلاس یازدهم A

۱- الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}\geq 0$

مخرج مشترک: $\frac{-2x+1}{x(x+1)}\geq 0$

$\frac{x-1}{x}\geq\frac{x}{x-1}$ $\quad (x-1)^2\geq x^2$ $\quad x^2-2x+1\geq x^2$ $\quad -2x+1\geq 0$ $\quad -2x\geq -1$ $\quad 2x\leq 1$ $\quad \boxed{x\leq\frac{1}{2}}$

$D_f=(-\infty,\frac{1}{2}]-\{0\}$

$(-\infty,0)\cup[\frac{1}{2},1)$

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ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$

دامنه: ریشه‌های مخرج $-R$

$x+1\neq 0$
$x\neq 0$
$x+2\neq 0$
$x-1\neq 0$

$\frac{3}{x-1}+\frac{1}{x+2}\neq 0$ $\quad x\neq -\frac{5}{4}$

$D_f=\mathbb{R}-\{-2,-1,0,1,-\frac{5}{4}\}$

۲- الف) $f(x)=\sqrt{\left((\frac{1}{3})^x-9\right)(32-2^x)}$

$(\frac{1}{3})^x-9=0 \quad x=-2$

$32-2^x=0 \quad x=5$

$D_f=(-\infty,-2]\cup[5,+\infty)$

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ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt[4]{x-1}=3-\sqrt{y+1}$ $\quad \boxed{x\geq 1}$

$3-\sqrt{y+1}\leq 3 \Rightarrow \sqrt[4]{x-1}\leq 3$

$x-1\leq 81$ $\quad \boxed{x\leq 82}$

$\boxed{D_f=[1,82]}$

۳- $f(x)=\dfrac{\log_2(x^2-x-2)}{\sqrt{x^2-1}+1}$

$x^2-x-2>0$
$(x-2)(x+1)>0$

$x>2 \quad x<-1$

$x^2-1\geq 0 \quad x^2\geq 1$

$\boxed{x\leq -1}$ $\boxed{x\geq 1}$ $\cap$

$D_f=(-\infty,-1)\cup(2,+\infty)$

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۴- $\sqrt{3+ax-x^2}=\sqrt{-(x^2-ax-3)}$

$(x+2)(x-\frac{3}{2})$

$(x+2)(-x+\frac{3}{2})=-x^2-\frac{1}{2}x+3$

$\boxed{a=-\frac{1}{2}}$ $\boxed{b=\frac{3}{2}}$

$a+b=\frac{3}{2}-\frac{1}{2}=\frac{2}{2}=\boxed{1}$

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۵- $f(x)=\begin{cases}3x-3 & : x\geq 1\\ 2x+3 & : x<1\end{cases}$ $\quad g(x)=\sqrt{f(x)-x}$

$\begin{cases}x\geq 1 & : 3x-2-x=2x-2\\ x<1 & : 2x+3-x=x+3\end{cases}$

$2x-2\geq 0 \quad 2x\geq 2 \quad \boxed{x\geq 1}$

$x+3\geq 0 \quad x\geq -3 \quad -3\leq x<1$

$D_f=[-3,+\infty)$

۲

-6

$$f(x)=\begin{cases}(a+1)(x+2) & : x>1\\ 2a+2x & : x\leq 1\end{cases}$$

$2f(5)=f(-2)+a$

$f(5)=(a+1)(5+2)=7a+7$

$f(-2)=2a+-4=3a-4$

$2(7a+7)=3a-4+a$

$14a+14=2a-4+a$

$16a=-18 \quad a=-\frac{18}{16}=\boxed{-\frac{9}{8}}$

دقت!! $10a=-18 \rightarrow a=-1.8$

1.5

-7

$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$

$f(2-\sqrt{3})+f(2+\sqrt{3})=2\sqrt{6}+4$ ✓

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$f(2-\sqrt{3})=\sqrt{2-\sqrt{3}}+\frac{1}{\sqrt{2-\sqrt{3}}}+2 \quad (+) \quad f(2+\sqrt{3})=\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}+2$

$\left(\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}\right)+\left(\frac{1}{\sqrt{2-\sqrt{3}}}+\frac{1}{\sqrt{2+\sqrt{3}}}\right)+4$

$\rightarrow \left(\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}\right)^2=2-\sqrt{3}+2+\sqrt{3}+2\sqrt{(2-\sqrt{3})(2+\sqrt{3})}=4+2=6 \Rightarrow \sqrt{6}$

$\frac{1}{\sqrt{2-\sqrt{3}}}+\frac{1}{\sqrt{2+\sqrt{3}}}=\frac{\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}}{\sqrt{(2+\sqrt{3})(2-\sqrt{3})}}=\frac{\sqrt{6}}{1}=\boxed{\sqrt{6}} \rightarrow -$

$4-3=1$

-8

1.75

$\times 2 \quad 2f(x)-3f(-x)=4x^2-x \qquad 4f(x)-6f(-x)=8x^2-2x$

$\times 3 \quad 2f(-x)-3f(x)=4x^2+x \qquad 6f(-x)-9f(x)=12x^2+3x$

$-5f(x)=20x^2+x$

$f(x)=\frac{20x^2+x}{-5}=-4x^2-\frac{x}{5}$

-9

$(x+2)f(x)-2xf(x+2)=2x^3-mx+3m-1$

$2f(0)=(3\times 3)+1=10 \qquad f(0)=\cancel{5}\ \frac{25}{4}$

$x=0 \Rightarrow 2f(0)=3m-1 \xrightarrow{\times 3} 6f(0)=9m-3$

$x=-2 \Rightarrow (0)f(-2)+6f(0)=16+2m+3m-1=15+5m$

$9m-3=15+5m$

$4m=18$

$m=\cancel{5}\ 4.5$

1

-10

$f(-1)+f(\frac{1}{-1})=2f(-1)=\frac{3+12+3}{-1}=-18 \qquad f(-1)=\boxed{-9}$ ✓

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$x f(x)+f(\frac{1}{x})=3x-12+\frac{3}{x} \Rightarrow f(x)=ax+b \ (a\rightarrow 3,\ b\rightarrow -6) \Rightarrow f(x)=3x-6$

$3(-1)-6=\boxed{-9}$