باسمه تعالی

نام و نام خانوادگی: علی ربانی — پاسخنامه تکلیف شماره ۴ — کلاس یازدهم A

۱- الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$

$$D_f=(-\infty,\tfrac{1}{2}]-\{0\}$$

$\frac{x-1}{x}\geq\frac{x}{x-1}$ $\quad (x-1)^2\geq x^2$

$x^2-2x+1\geq x^2 \quad -2x+1\geq 0 \quad -2x\geq -1 \quad 2x\leq 1 \quad \boxed{x\leq\frac{1}{2}}$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+3}}$

دامنه ریشه‌های مخرج $R-$

$x+1\neq 0$

$x\neq 0$

$x+3\neq 0$

$x-1\neq 0$

$$D_f=\mathbb{R}-\{-3,-1,0,1\}$$

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۲- الف) $f(x)=\sqrt{\left((\frac{1}{3})^x-9\right)\left(32-4^x\right)}$

$$\boxed{D_f=(-\infty,-2]\cup[\tfrac{5}{2},+\infty)}$$

$(\frac{1}{3})^x-9=0 \quad x=-2$

$32-4^x=0 \quad x=\frac{5}{2}$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$3-\sqrt{y+1}\leq 3 \Rightarrow \sqrt[4]{x-1}\leq 3$

$\sqrt[4]{x-1}=3-\sqrt{y+1}$

$x-1\leq 81$

$\boxed{x\leq 82}$

و $\boxed{x\geq 1}$

$$\boxed{D_f=[1,82]}$$

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۳- $f(x)=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$

$x^2-x-2>0$

$(x-2)(x+1)>0$

$$\boxed{D_f=(-\infty,-1)\cup(2,+\infty)}$$

$x>2 \quad x<-1$

$x^2-1\geq 0 \quad x^2\geq 1$

$\boxed{x\leq -1} \quad \boxed{x\geq 1} \quad \cap$

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۴- $\sqrt{3+ax-x^2}=\sqrt{-(x^2-ax-3)}$

$(x+2)(x-\frac{3}{2})$

$(x+2)(-x+\frac{3}{2})=-x^2-\frac{1}{2}x+3$

$\boxed{a=-\frac{1}{2}}$ $\quad \boxed{b=\frac{3}{2}}$

$a+b=\frac{3}{2}-\frac{1}{2}=\frac{2}{2}=\boxed{1}$

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۵- $f(x)=\begin{cases}3x-2 & : x\geq 1\\ 2x+3 & : x<1\end{cases}$ $\qquad g(x)=\sqrt{f(x)-x}$

$$\boxed{D_f=[-3,+\infty)}$$

$\begin{cases}x\geq 1: & 3x-2-x=2x-2\\ x<1: & 2x+3-x=x+3\end{cases}$

$2x-2\geq 0 \quad 2x\geq 2 \quad \boxed{x\geq 1}$

$x+3\geq 0 \quad x\geq -3 \quad -3\leq x<1$

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$$f(x)=\begin{cases}(a+1)(x+2) & : x>1\\ 2a+2x & : x\leq 1\end{cases}$$

$$\underbrace{2f(5)}=f(-3)+a$$

$$f(5)=(a+1)(5+2)=7a+7$$

$$f(-3)=2a+-6=3a-4$$

$$2(7a+7)=3a-4+a$$

$$14a+14=2a-4+a$$

$$16a=-18 \quad a=-\frac{18}{16}=\boxed{-\frac{9}{8}}$$

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$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \qquad \boxed{f(2-\sqrt{3})+f(2+\sqrt{3})=2\sqrt{6}+4}$$

$$f(2-\sqrt{3})=\sqrt{2-\sqrt{3}}+\frac{1}{\sqrt{2-\sqrt{3}}}+2 \;\oplus\; f(2+\sqrt{3})=\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}+2$$

$$\left(\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}\right)+\left(\frac{1}{\sqrt{2-\sqrt{3}}}+\frac{1}{\sqrt{2+\sqrt{3}}}\right)+4$$

$$\rightarrow\left(\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}\right)^2=2-\sqrt{3}+2+\sqrt{3}+2\sqrt{(2-\sqrt{3})(2+\sqrt{3})}=4+2=6\Rightarrow\sqrt{6}$$

$$\frac{1}{\sqrt{2-\sqrt{3}}}+\frac{1}{\sqrt{2+\sqrt{3}}}=\frac{\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}}{\sqrt{(2+\sqrt{3})(2-\sqrt{3})}}=\frac{\sqrt{6}}{1}=\boxed{\sqrt{6}}\rightarrow -$$

$$4-3=1$$

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$$\begin{cases}2f(x)-3f(-x)=4x^2-x & \times 2\\ 2f(-x)-3f(x)=4x^2+x & \times 3\end{cases}$$

$$4f(x)-6f(-x)=8x^2-2x$$

$$6f(-x)-9f(x)=12x^2+3x$$

$$-5f(x)=20x^2+x$$

$$f(x)=\frac{20x^2+x}{5}=\boxed{4x^2+\frac{x}{5}}$$

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$$(x+2)f(x)-2xf(x+2)=4x^2-mx+3m-1$$

$$x=0\Rightarrow 2f(0)=3m+1\xrightarrow{\times 3}6f(0)=9m+3$$

$$x=-2\Rightarrow (0)f(-2)+6f(0)=16+2m+3m-1=15+5m$$

$$9m+3=15+5m$$

$$4m=12$$

$$\underline{m=3}$$

$$2f(0)=(3\times3)+1=10 \qquad \boxed{f(0)=5}$$

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$$f(-1)+f\left(\frac{1}{-1}\right)=2f(-1)=\frac{3+12+3}{-1}=-18 \qquad f(-1)=\boxed{-9}$$

$$\text{یا}\quad f(x)+f\left(\frac{1}{x}\right)=3x-12+\frac{3}{x}\Rightarrow f(x)=\underset{\downarrow\,3}{a}x+\underset{\downarrow\,-6}{b}\Rightarrow f(x)=3x-6$$

$$3(-1)-6=\boxed{-9}$$