

الف) $\sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0 \Rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$

$\Rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{-2x+1}{x(x-1)} \geq 0 \rightarrow \frac{0}{+} \frac{+}{-} \frac{+}{+} \frac{-}{-} \Rightarrow Df = (-\infty, 0) \cup [\frac{1}{2}, +\infty)$

ب) $\{R - \{-2, -1\}\}$ (خرج کویکلاف صفر)
 $\{ \frac{3}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{3x+2+x-1}{(x-1)(x+2)} \neq 0 \Rightarrow 4x+1 \neq 0 \Rightarrow x \neq -\frac{1}{4} \}$ (خرج کویکلاف صفر)
 $\Rightarrow Df = R - \{-2, -1, -\frac{1}{4}\}$

الف) $(\frac{1}{x})^x - 9 \geq 0 \Rightarrow \frac{1}{x} - 9 \geq 0 \Rightarrow \frac{-2}{+} \frac{2/0}{-} \Rightarrow Df = (-\infty, -2] \cup [2/0, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{x-1} + y+1 = 9 \Rightarrow y = 8 - \sqrt{x-1}$

$y \geq -1 \Rightarrow 8 - \sqrt{x-1} \geq -1 \Rightarrow \sqrt{x-1} \leq 9 \Rightarrow x-1 \leq 81 \Rightarrow x \leq 82$
 $x \geq -1 \Rightarrow Df = [-1, 82]$

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \begin{cases} x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow \frac{-1}{+} \frac{2}{-} \Rightarrow \text{①} \\ x^2 - 1 > 0 \Rightarrow (x-1)(x+1) > 0 \Rightarrow \frac{-1}{+} \frac{1}{-} \Rightarrow \text{②} \\ \sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1 \end{cases}$

$\Rightarrow Df = \text{①} \cap \text{②} = (-\infty, -1) \cup (2, +\infty)$

$\sqrt{3+an-x^2} \geq 0 \Rightarrow -(x^2 - ax - 3) \geq 0 \Rightarrow \frac{-a}{-} \frac{b}{+}$

$\Rightarrow -(x+2)(x-\frac{3}{2}) \geq 0 \Rightarrow -(x^2 + \frac{1}{2}x - 3) = -x^2 - \frac{1}{2}x + 3 = -x^2 + ax + b$

$\Rightarrow a = -\frac{1}{2} \text{ و } b = 3 \Rightarrow a+b = \frac{5}{2} = 2.5$

$g(x) = \sqrt{f(x) - x} \Rightarrow f(x) - x \geq 0 \Rightarrow f(x) \geq x$

$\Rightarrow \begin{cases} 3x - 2 \geq x \Rightarrow 2x - 2 \geq 0 \Rightarrow \frac{-2}{-} \frac{0}{+} \Rightarrow [1, +\infty) \text{ ①} \\ 2x + 3 \geq x \Rightarrow x + 3 \geq 0 \Rightarrow \frac{-3}{-} \frac{0}{+} \Rightarrow (-\infty, 3] \text{ ②} \end{cases}$

$\Rightarrow Dg = \text{①} \cap \text{②} = [-3, 3] \cup [1, +\infty) = [-3, +\infty)$

$$2f(a) = f(-1) + a \Rightarrow 2(a+1)(\frac{0+2}{1}) = 3a - 1 + a$$

$$\Rightarrow 4a + 4 = 4a - 1 \Rightarrow 1 \cdot a = -1 \Rightarrow a = \frac{-1}{1} = -1 \quad \checkmark$$

$$(2-\sqrt{3})(2+\sqrt{3}) = 1 \xrightarrow{\text{ریشه ها معکوس}} 2-\sqrt{3} = a, 2+\sqrt{3} = \frac{1}{a}$$

$$\Rightarrow f(a) + f(\frac{1}{a}) = \sqrt{a} + \frac{1}{\sqrt{a}} + 2 + \frac{1}{\sqrt{a}} + \sqrt{a} + 2 = 2(\sqrt{a} + \frac{1}{\sqrt{a}}) + 4$$

$$\Rightarrow (\sqrt{a} + \frac{1}{\sqrt{a}})^2 = a + \frac{1}{a} + 2 \Rightarrow \sqrt{a} + \frac{1}{\sqrt{a}} = \sqrt{4} \Rightarrow 2\sqrt{4} = 4 \quad \checkmark$$

$$\begin{aligned} 2f(x) - 3f(-x) &= 5x^2 - 2x & 4f(x) - 9f(-x) &= 12x^2 - 2x \\ 2f(-x) - 3f(x) &= 5x^2 + 2x & 4f(-x) - 9f(x) &= 12x^2 + 2x \end{aligned}$$

$$\Rightarrow -8f(x) = 2 \cdot x^2 + 2x \Rightarrow f(x) = -\frac{2 \cdot x^2 + 2x}{8} \quad \checkmark$$

$$(x+2)f(x) - 3xf(x+2) = 5x^2 - mx + 3m - 1$$

$$\stackrel{x=-2}{\Rightarrow} 4f(0) = 1 + 2m + 3m - 1 \Rightarrow 4f(0) = 5m \quad \checkmark$$

$$\stackrel{x=0}{\Rightarrow} 2f(0) = 3m - 1 \quad \checkmark \Rightarrow 4f(0) = 6m - 2$$

$$\Rightarrow 6m - 2 = 5m \Rightarrow m = 2 \quad \checkmark \Rightarrow f(0) = \frac{5m - 1}{4} = \frac{10 - 1}{4} = \frac{9}{4}$$

$$\stackrel{x=-1}{\Rightarrow} f(-1) + f(-1) = \frac{3(-1)^2 - 1(-1) + 3}{-1} = \frac{3 + 1 + 3}{-1} = -1 \quad \checkmark$$

$$\Rightarrow \underline{f(-1) = -1} \quad \checkmark$$